Homework Set 3

Due: October 21, 2008, *before class*

1. **Neutrinos**

Consider the following neutrino fluxes and energies from the sun:

Table 8.3 Properties of solar neutrinos

Source	*Flux at Earth* *(m^{-2} s^{-1})*	*Energy* *(MeV)*	*Average* *(MeV)*
$p + p \rightarrow {}^2\mathrm{D} + e^+ + \nu$	6.0×10^{14}	≤ 0.42	0.263
${}^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu$	4.9×10^{13}	0.86 (90%), 0.38 (10%)	0.80
${}^8\mathrm{B} \rightarrow {}^8\mathrm{Be} + e^+ + \nu$	5.7×10^{10}	≤ 15	7.2

(a) **Based on these fluxes, what is the neutrino number flux at the surface of the sun per** cm^2**?** Compute the flow through a surface perpendicular to the earth sun connection line.

Neutrino flux

$$F = (6.0\times10^{14} + 4.9\times10^{13} + 5.7\times10^{10})\,\mathrm{m^{-2}\,s^{-1}} \times \left(\frac{1\,\mathrm{A.U.}}{1\,\mathrm{R_\odot}}\right)^2$$

$$\ldots = 6.49057\times10^{14}\,\mathrm{m^{-2}\,s^{-1}} \times \left(\frac{1.4959787\times10^{13}\,\mathrm{cm}}{6.98\times10^{10}\,\mathrm{cm}}\right)^2$$

$$\ldots = 2.9814176\times10^{15}\,\mathrm{cm^{-2}\,s^{-1}} \approx 3\times10^{15}\,\mathrm{cm^{-2}\,s^{-1}}$$

Score: **4**

(b) **Based on the fluxes table above, what is energy flux ($\mathrm{erg\,cm^{-2}\,s^{-1}}$) at the surface of the sun?**

$1\,\mathrm{MeV} = 1.6021\times10^{-6}\,\mathrm{erg}$

Energy flux

$$J = (6.0\times10^{10} \times 0.263 + 4.9\times10^{9} \times 0.80 + 5.7\times10^{6} \times 7.2)\,\mathrm{MeV\,cm^{-2}\,s^{-1}} \times \left(\frac{1\,\mathrm{A.U.}}{1\,\mathrm{R_\odot}}\right)^2$$

$$\ldots = 1.974104\times10^{10}\,\mathrm{MeV\,cm^{-2}\,s^{-1}} \times \left(\frac{1.4959787\times10^{13}\,\mathrm{cm}}{6.98\times10^{10}\,\mathrm{cm}}\right)^2$$

$$\ldots = 1.453\times10^{9}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$$

Score: **4**

(c) **What is energy flux ($\mathrm{erg\,cm^{-2}\,s^{-1}}$) at the surface of the earth?**

$1\,\mathrm{MeV} = 1.6021\times10^{-6}\,\mathrm{erg}$

Energy flux

$$J = (6.0\times10^{10} \times 0.263 + 4.9\times10^{9} \times 0.80 + 5.7\times10^{6} \times 7.2)\,\mathrm{MeV\,cm^{-2}\,s^{-1}}$$

$$\ldots = 1.974104\times10^{10}\,\mathrm{MeV\,cm^{-2}\,s^{-1}} = 3.1638\times10^{4}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$$

Score: **2**

(d) **How many solar neutrinos are on average in a box of $1\,\mathsf{cm}^3$ on the surface of the earth at any given time?**
Assume neutrinos move at the speed of light.

$$N = \frac{F}{c} = \frac{6.5\times10^{10}}{2.998\times10^{10}}\,\mathsf{cm}^{-3} = 2.165\,\mathsf{cm}^{-3} \approx 2\,\mathsf{cm}^{-3}$$

Score: **2**

(e) **What is the energy density from neutrinos (in $\mathsf{erg}/\mathsf{cm}^3$) at the surface of the earth?**

$$\epsilon = \frac{J}{c} = \frac{3.1638\times10^{4}}{2.998\times10^{10}}\,\mathsf{erg\,cm}^{-3} = 1.0553\times10^{-6}\,\mathsf{erg\,cm}^{-3} \approx \frac{1}{1,000,000}\,\mathsf{erg\,cm}^{-3}$$

Score: **2**

2. **Eddington Limit.**
We now know that some galaxies host "super-massive" black holes of some $10^9\,\mathsf{M}_\odot$ in their center. Let us make a few simple assumptions

- They accrete fully ionized pure hydrogen ($^1\mathsf{H}$) gas.
- They accrete at $10\,\%$ of the Eddington accretion rate.
- The accreted matter releases $1\,\%$ of its rest mass as "accretion luminosity".

Using

$$L_{\mathsf{edd}} = \frac{4\pi c G M \mathsf{u}}{\sigma_\mathsf{T}}$$

$$L_{\mathsf{acc}} = 0.01\,\dot{M}c^2$$

we set

$$L_{\mathsf{acc}} = 0.1\,L_{\mathsf{edd}}$$

and solve for $\dot{M}$

$$\dot{M} = \frac{0.1\,L_{\mathsf{edd}}}{0.01\,c^2} = 10\,\frac{L_{\mathsf{edd}}}{c^2} = M\,10\,\frac{4\pi G\mathsf{u}}{\sigma_\mathsf{T}\,c} = \nu\,M$$

where we define

$$\nu = \frac{40\pi G\mathsf{u}}{\sigma_\mathsf{T}\,c} = 6.98\times10^{-16}\,\mathsf{s}^{-1} \approx 2.20\times10^{-8}\,\mathsf{yr}^{-1}$$

(a) **How long does it take for an initial $10\,\mathsf{M}_\odot$ stellar black to double its mass?**

$$\dot{M} = \frac{\mathsf{d}M}{\mathsf{d}t} = \nu\,M \quad\Rightarrow\quad M(t) = M_0\,e^{\nu(t-t_0)} \quad\Rightarrow\quad t - t_0 = \ln\left(\frac{M}{M_0}\right)\nu^{-1}$$

Using $M(t_0) = 10\,\mathsf{M}_\odot$, $M(t) = 20\,\mathsf{M}_\odot$ we have

$$20\,\mathsf{M}_\odot = 10\,\mathsf{M}_\odot\,e^{\nu(t_1-t_0)} \quad\Rightarrow\quad e^{\nu(t_1-t_0)} = 2 \quad\Rightarrow\quad \nu(t_1-t_0) = \ln 2$$

$$t_1 - t_0 = \frac{\ln 2}{\nu} = 9.93\times10^{14}\,\mathsf{s} = 3.145\times10^{7}\,\mathsf{yr}$$

Score: **4**

(b) **How long does it take for an initial $20\,\mathsf{M}_\odot$ stellar mass black hole to double its mass?**
The same time. This is obvious from

$$t - t_0 = \ln\left(\frac{M}{M_0}\right)\nu^{-1}$$

as we have the same ratio M/M_0. Score: **2**

(c) **How long does it take for an initial $10\,\mathsf{M}_\odot$ stellar black to grow to $10^9\,\mathsf{M}_\odot$?**

Again, using

$$t - t_0 = \ln\left(\frac{M}{M_0}\right)\nu^{-1}$$

with $M/M_0 = 10^8$ we have

$$t - t_0 = \ln\left(10^8\right)\nu^{-1} = 2.63\times10^{16}\,\mathsf{s} \approx 8.37\times10^8\,\mathsf{yr}$$

Score: **2**

(d) **How long does it take for an initial $10\,\mathsf{M}_\odot$ stellar black to grow to $10^9\,\mathsf{M}_\odot$ if it accretes mass at $100\,\%$ Eddington Luminosity?**

$10\,\%$ of the that time.
Due to the $10\times$ higher efficiency, we replace ν by $10\,\nu$ and have

$$t - t_0 = \ln\left(10^8\right)(10\,\nu)^{-1} = 2.63\times10^{15}\,\mathsf{s} \approx 8.37\times10^7\,\mathsf{yr}$$

Score: **2**

3. **Neutron Star Energy**

Assume a neutron stars has a "gravitational mass" ((rest mass) - (binding energy)/c^2) of $1.4\,\mathsf{M}_\odot$ and that the binding energy is $20\,\%$ of its rest mass. Almost all of this energy is released in the form of neutrinos when the neutron star forms from the collapsing iron core of a massive star.

(a) **What is the rest mass of the neutron star?**

$$(\text{binding energy})/c^2 = 0.2\,(\text{rest mass})$$

$$((\text{rest mass}) - 0.2\,(\text{rest mass})) = 0.8\,(\text{rest mass}) = 1.4\,\mathsf{M}_\odot$$

$\Rightarrow$

$$(\text{rest mass}) = \frac{1.4\,\mathsf{M}_\odot}{0.8} = 1.75\,\mathsf{M}_\odot$$

Score: **2**

(b) **How much energy is released during neutron star formation (binding energy)?**

$$E_{\mathsf{bind}} = 0.2 \times 1.75\,\mathsf{M}_\odot \times c^2 = 6.26\times10^{53}\,\mathsf{erg} = 6.26\times10^{46}\,\mathsf{J}$$

Score: **2**

(c) Assume the energy is radiated away as neutrinos with energy of $5\,\mathsf{MeV}$.
How many neutrinos are emitted during neutron star formation under these assumptions?

Using

$$1\,\mathsf{eV} = 1.6021773\times10^{-12}\,\mathsf{erg}\,, \quad 1\,\mathsf{MeV} = 1.6021773\times10^{-6}\,\mathsf{erg}$$

we obtain for the number of neutrinos

$$n = \frac{E_{\mathsf{bind}}}{5\,\mathsf{MeV}} = 7.79\times10^{58}$$

Score: **2**

(d) Assume these neutrinos are emitted on a time scale of $3\,\mathsf{s}$. For simplicity, assume a constant rate. **At what distance of the neutron star from Earth is the neutrino flux from the neutron star the same as the solar neutrino flux (in neutrinos per cm^2).**

We use a solar neutrino flux of $F_\odot = 6.5\times10^{10}\,\mathsf{cm}^{-2}\,\mathsf{s}^{-1}$. The release time is $\tau = 3\,\mathsf{s}$, the total number of neutrinos released is $n = 7.79\times10^{58}$ ($\approx 2.6\times10^{58}$ neutrinos per second). The supernova neutrino flux as a function of distance d is

$$F_{\mathsf{SN}}(d) = \frac{n}{\tau}\frac{1}{4\pi d^2}$$

Setting these two equal and solving for d gives

$$d = \left(\frac{n}{4\pi\tau F_\odot}\right)^{1/2} = 1.78\times10^{23}\,\mathsf{cm} = 1.89\times10^{5}\,\text{light years}$$

Score: **4**

(e) When the neutron star forms, it converts protons into neutrons by electron capture, $(\mathsf{e}^-,\nu_{\mathsf{e}})$, releasing an electron neutrino. For simplicity, assume that initially half the baryons are protons, and that all of them are converted to neutrons.

How many electron neutrinos are emitted during neutron star formation under these assumptions?

The total rest mass is, as before $1.75\,\mathsf{M}_\odot$, half of these are protons with a mass of about $1\,\mathsf{u}$. The total number of electron neutrinos released by this process is the same as the number of protons. This number is

$$n = \frac{1.75\,\mathsf{M}_\odot}{2\,\mathsf{u}} = 1.05\times10^{57}$$

Therefore, this is only a small effect compared to the number of neutrinos released from binding energy.

Score: **4**