Homework Set 6

Due: December 2, 2008, *before class*

1. **Star Formation.**

   (a) **What is the Jeans mass for cloud of molecular hydrogen of density $10^{-22}\,\mathsf{g\,cm^{-3}}$ and temperature $10\,\mathsf{K}$, assuming a value of $1/2$ for α in equation (10.5) of Prialnik (ratio of actual binding energy of the cloud relative to GM^2/R)?**

   Molecular hydrogen has $\mu = 2$, $\mathcal{R} = 8.3145\times10^7\,\mathsf{erg\,K^{-1}}$ is the gas constant, $G = 6.67\times10^{-8}\,\mathsf{cm^3\,g^{-1}\,s^{-1}}$ is the gravitational constant:

$$M_{\mathsf{Jeans}} = \left[\left(\frac{3}{4\pi}\right)^{1/2}\left(\frac{3}{\alpha}\right)^{3/2}\right]\left(\frac{\mathcal{R}T}{\mu G}\right)\rho^{-1/2} = 3.533\times10^{24}\times\rho^{-1/2} = 3.533\times10^{35}\,\mathsf{g} = 177.6\,\mathsf{M_\odot}$$

   or when using the approximate formula one obtains

$$M_{\mathsf{Jeans}} \approx 10^2\,\mathsf{M_\odot}\left(\frac{T^3}{n}\right)^{1/2} = 182.6\,\mathsf{M_\odot}$$

   NOTE: The above formula is adopted to use cgs units; in Prialnik the number density is given in m^{-3} not $\mathsf{cm^{-3}}$. Score: **2**

   (b) **Compute α for a spherical gas could of constant density.**

   Let R be the radius of the cloud and M its mass. Using

$$\rho = \frac{3M}{4\pi R^3}\ , \quad \mathsf{d}m = 4\pi r^2\rho\mathsf{d}r\ , \quad m(r) = \frac{4\pi}{3}r^3\rho$$

   We can write Prialnik's equation 2.20 as

$$\Omega = -\int_0^M \frac{G}{r}m\mathsf{d}m = -\int_0^R \frac{G}{r}\frac{4\pi}{3}r^3\rho 4\pi r^2\rho\mathsf{d}r = -\frac{(4\pi\rho)^2 G}{3}\int_0^R r^4\mathsf{d}r$$

$$\ldots = -\frac{(4\pi)^2 G}{3}\left(\frac{3M}{4\pi R^3}\right)^2\frac{1}{5}R^5 = -\frac{3}{5}\frac{GM^2}{R}$$

   That is, $\alpha = 3/5$. Obviously, this is a lower limit for α for all realistic stellar configurations with monotonously decreasing density. Score: **6**

   (c) **By what factor does the Jeans mass change if the cloud is compressed by a factor 2 in density but temperature remains the same (isothermal compression).**

   $M_{\mathsf{Jeans}} \propto \rho^{-1/2}$ therefore the Jeans mass decreases by a factor $1/\sqrt{2}$, that is, is smaller by a factor 0.7071. Score: **1**

   (d) The adiabatic index is given by $\gamma = C_P/C_V$ and is $5/3$ for a mono-atomic gas and $7/5$ for a diatomic gas. For adiabatic compression $T\rho^{1-\gamma}$ remains constant.

   By what factor does the Jeans mass change for a cloud of molecular hydrogen if the cloud is adiabatically compressed by a factor 2 in density?

   Molecular hydrogen is a diatomic gas, hence we use $\gamma = 7/5$. From the condition of adiabaticity we obtain

$$T \propto \rho^{\gamma-1} = \rho^{2/5}$$

and for the Jeans mass we hence have

$$M_{\mathsf{Jeans}} \propto \left(\frac{T^3}{\rho}\right)^{1/2} \propto \left(\frac{(\rho^{2/5})^3}{\rho}\right)^{1/2} \propto \rho^{1/10}$$

If the gas cloud is compressed adiabatically by a factor 2, that is, its density increased by factor 2, the Jeans mass is changed by a factor $2^{0.1} = 1.07177$, that is, increases by 7.1 %. A cloud that just has its Jeans mass is stable under such compressions as long as $\gamma > 4/3$. Score: **3**

Please use cgs units for calculations and numerical values.