Astrophysics I: Stars and Stellar Evolution

AST 4001

Alexander Heger[1,2,3]

[1] School of Physics and Astronomy
University of Minnesota

[2] Theoretical Astrophysics Group, T-6
Los Alamos National Laboratory

[3] Department of Astronomy and Astrophysics
University of California at Santa Cruz

Stars and Stellar Evolution, Fall 2008

Agenda

1. Recap
   - Web site access
   - The Sun
   - Stellar Systems
2. Equations of Stellar Evolution
   - Energy Equation
3. Summary
   - Stellar Structure equations
   - Feedback
   - Build Your Own Star

Overview

1. Recap
   - Web site access
   - The Sun
   - Stellar Systems
2. Equations of Stellar Evolution
   - Energy Equation
3. Summary
   - Stellar Structure equations
   - Feedback
   - Build Your Own Star

Contact

- **Location & Dates:**
  Physics 236A, MTWTh 10:10-11:00 AM
- **Office hours:**
  Wednesdays, 13:00-14:30, 342F Tate
- **email:**
  I cannot guarantee that I will receive all emails due to SPAM filters. On class days I will try to reply to email within 24 h.
- **Web site:**
  `http://stellarevolution.org/AST-4001`
  I will post notes, updates, problem sets, etc.
- **Google course calendar** (on Web site):
  `o86pe6r5paic30h4qv6acm9ej0%40group.calendar.google.com`

Web site access

- user name: **Ast-4001**
- password: **&32y^nbY**

Solar Convection (3D simulation)

(solar convection)

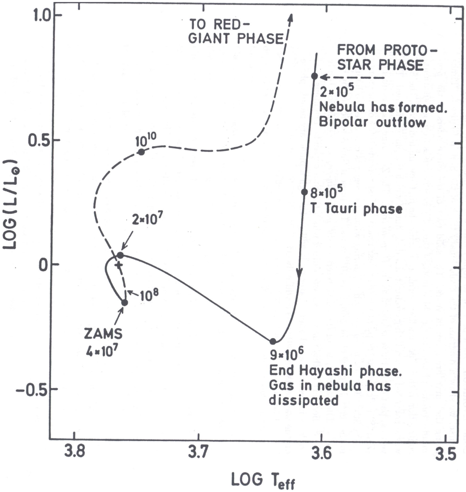
Solar Convection

(solar convection)

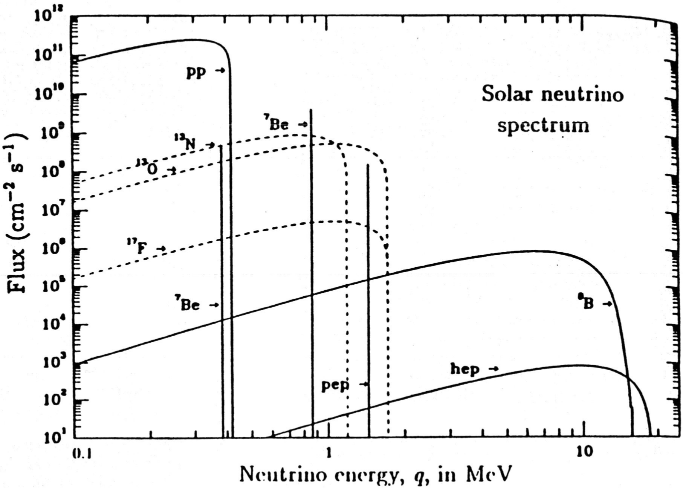
Solar Convection

(solar convection)

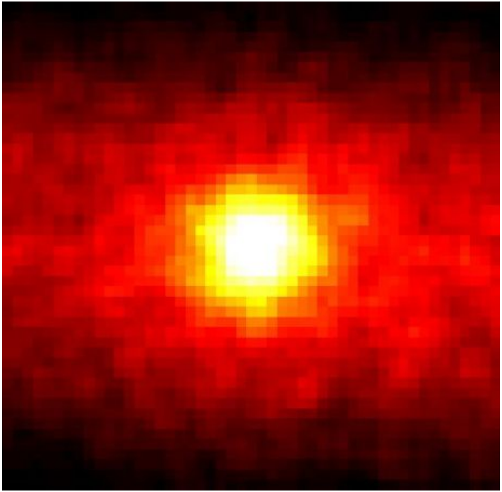
Evolution of the Sun in the HRD



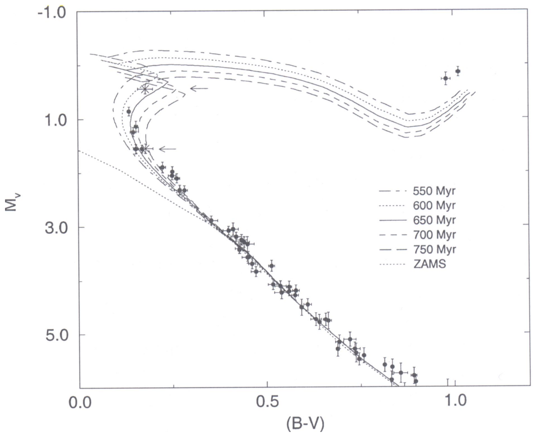
The Solar Neutrino Spectrum



The Sun as Seen in Neutrinos

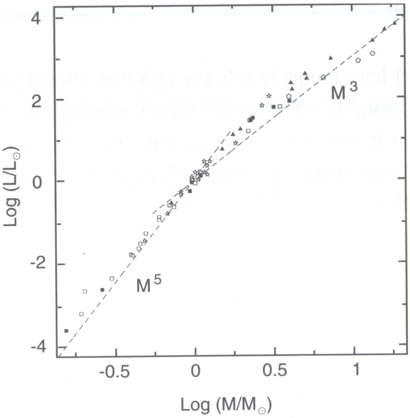


Cluster Ages



Hyades cluster and stellar tracks

Mass-Luminosity Relation



Mass-Luminosity relation for (zero-age) main-sequence (ZAMS) stars

$$L \propto M^{\nu}$$

with $\nu = 3 \ldots 5$.
Can be calibrated piecewise to

$$\frac{L}{\mathrm{L}_\odot} = \left(\frac{M}{\mathrm{M}_\odot}\right)^{\nu}$$

Overview

1. Recap
   - Web site access
   - The Sun
   - Stellar Systems
2. Equations of Stellar Evolution
   - Energy Equation
3. Summary
   - Stellar Structure equations
   - Feedback
   - Build Your Own Star

Stellar Structure Equations

stationary terms time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad (1)$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \quad (2)$$

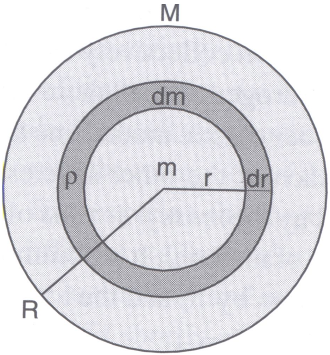
$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \quad (3)$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \quad (4)$$

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) \quad (5)$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Relation between mass and radius



- integral formulation:

$$m(r) = \int_0^r 4\pi r^2 \rho(r)\,\mathrm{d}r$$

- differential formulation

$$\mathrm{d}m = 4\pi\rho r^2 \mathrm{d}r$$

Stellar Gas

- stellar "gas" composed ions, electron, and radiation
- radiation regarded as "photon gas" with quanta caring $h\nu$ energy and $h\nu/c$ momentum
- photon gas described by Planck spectrum
- ion/electron gas described by Maxwellian velocity distribution
- at high density and low temperature electron gas follows *degenerate* equation of state (Fermi statistics)
- at even lower T and higher ρ ions (nucleons) can be degenerate (e.g., neutron stars)

Local Thermodynamic Equilibrium (LTE)

In good approximation in stars:

- *mean free path* is much smaller than the size of the system, or the scale on which the properties change inside the system
- *mean free time* is much smaller than the size of the system, or the scale on which the properties change inside the system
- frequent collisions between the different constituents
  $\rightsquigarrow$ local thermodynamical equilibrium (LTE)
  $\rightsquigarrow$ define unique *temperature* for all constituents

Structure and Variables

Assuming LTE allows to describe a spherical star of mass M uniquely at a given time t by

- $v(m, t)$ – (velocity)
- $\rho(m, t)$ – (density)
- $T(m, t)$ – (temperature)
- $X_i(m, t)$ – (composition - mass fraction)

If there are N species i, we require a minimum of $N + 2$ variables to describe the star.

Quiz

Why $N + 2$?

Discuss with your neighbor(s).

Structure and Variables

Mass fractions by definition sum up to 1:

$$\sum_{N=0}^{i} X_i = 1$$

This eliminates one independent variable.

NOTE: If $\rho(t, m)$ is know as a function of time, this would allow to eliminate the velocity equation as well.

In praxis of evolution codes, to obtain first order equation, we add

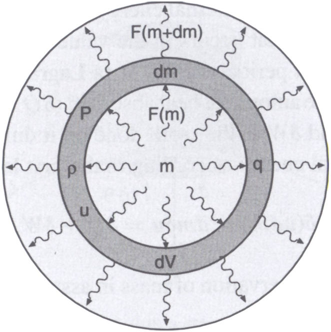
- $r(m,t)$ – (radius)
- $L(m,t)$ – (luminosity)

Variable r and m are interchangeable.

Lagrangian & Eulerian coordinates

- Eulerian coordinates:
  fixed in space
  variable r
- Lagrangian coordinates:
  co-moving with mass element
  variable m

Energy Equation



spherical shell inside star

$$\mathrm{d}V = 4\pi r^2\,\mathrm{d}r$$

$$\mathrm{d}m = \rho\,\mathrm{d}V = 4\pi r^2 \rho\,\mathrm{d}r$$

Using

- u – internal energy per unit mass
- P – pressure
- δt – small period of time
- δW – work on mass element during time δt
- δQ – heat absorbed by mass element during time δt

we can write

$$\delta(u\,\mathrm{d}m) = \mathrm{d}m\delta u = \delta Q + \delta W$$

Energy Equation

We can now write the work on the mass element δm as

$$\delta W = P\delta \mathrm{d}V = -P\delta\left(\frac{\mathrm{d}V}{\mathrm{d}m}\right)\,\mathrm{d}m = -P\delta\left(\frac{1}{\rho}\right)\,\mathrm{d}m$$

- compression means shrinking the of volume ($\delta \mathrm{d}V < 0$) and constitutes an addition of energy
- expansion means increasing the of volume ($\delta \mathrm{d}V < 0$) and constitutes an subtraction of energy

Energy Equation

Sources of *heat* are

- release of nuclear energy q
- (dissipation of other energy forms)
- balance of heat fluxes $F(m)$ from below and above

$$\delta Q = q\,\mathrm{d}m\delta t + F(m)\delta t - F(m+\mathrm{d}m)\delta t$$

Approximate

$$F(m+\mathrm{d}m) = F(m) + \left(\frac{\partial F}{\partial m}\right)\,\mathrm{d}m$$

to obtain

$$\delta Q = \left(q - \frac{\partial F}{\partial m}\right)\,\mathrm{d}m\,\delta t$$

Energy Equation

We can now substitute

$$\delta W = -P\delta\left(\frac{1}{\rho}\right)\,\mathrm{d}m \quad \text{and} \quad \delta Q = \left(q - \frac{\partial F}{\partial m}\right)\,\mathrm{d}m\,\delta t$$

into $\delta(u\,\mathrm{d}m) = \delta Q + \delta W$ and obtain

$$\delta(u\,\mathrm{d}m) + P\delta\left(\frac{1}{\rho}\right)\,\mathrm{d}m = \left(q - \frac{\partial F}{\partial m}\right)\,\mathrm{d}m\,\delta t$$

For $\delta t \longmapsto 0$ we obtain

$$\dot{u} + P\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = q - \frac{\partial F}{\partial m}$$

Energy Equation

In thermal equilibrium the time derivative vanishes:

$$q = \frac{\mathrm{d}F}{\mathrm{d}m}$$

Integrating over mass yields

$$\int_0^M q \, \mathrm{d}m = \int_o^M \mathrm{d}F = L$$

In most stars this energy is supplied by nuclear burning processes in the stellar interior. Generally we define the nuclear luminosity, L_{nuc} by

$$L_{\mathrm{nuc}} = \int_0^M q \, \mathrm{d}m$$

For a star in thermonuclear equilibrium with no other energy sources we hence have $L = L_{\mathrm{nuc}}$.

Overview

Summary

- Stellar interior described by Local thermodynamic equilibrium (LTE)
- $$\dot{u} + P \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = q - \frac{\partial F}{\partial m}$$
- $$L_{\mathrm{nuc}} = \int_0^M q \, \mathrm{d}m$$

Quick Write-up

Write a one-sentence summary of the most important point of today's lecture.

Stellar Evolution Project

- Bill Paxton's **EZ Stellar Evolution** code
  `http://www.kitp.ucsb.edu/~paxton/EZ-intro.html`
- Uses Linux `gfortran`
- g95 FORTRAN compiler can be downloaded for most platforms.
  `http://www.g95.org`