Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Agenda

1. Recap
   - Web site access
   - Energy Equation
2. Equations of Stellar Evolution
   - Equation of Motion
   - Equations of Composition Change
3. Summary
   - Stellar Structure equations
   - Build Your Own Star

Overview

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Contact

- **Location & Dates:**
  Physics 236A, MTWTh 10:10-11:00 AM
- **Office hours:**
  Wednesdays, 13:00-14:30, 342F Tate
- **email:**
  I cannot guarantee that I will receive all emails due to SPAM filters. On class days I will try to reply to email within 24 h.
- **Web site:**
  `http://stellarevolution.org/AST-4001`
  I will post notes, updates, problem sets, etc.
- **Google course calendar** (on Web site):
  `o86pe6r5paic30h4qv6acm9ej0%40group.calendar.google.com`

Web site access

- user name: **Ast-4001**
- password: **&32yˆnbY**

Energy Equation

- Stellar interior described by *local* thermodynamic equilibrium (LTE)
- energy equation

$$\dot{u} + P\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = q - \frac{\partial F}{\partial m}$$

- nuclear luminosity

$$L_{\mathrm{nuc}} = \int_0^M q \,\mathrm{d}m$$

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Stellar Structure Equations

stationary terms time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \tag{2}$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P\frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{3}$$

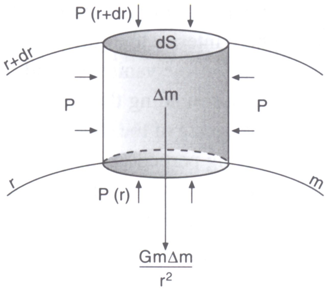
$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \tag{4}$$

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) \tag{5}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Force on Mass Element

small (cylindrical) volume



cross section: $\mathrm{d}S$
density: ρ
$\Rightarrow$ mass: $\Delta m = \rho\,\mathrm{d}r\,\mathrm{d}S$

Forces on mass element:

- gravitational force from sphere inside (below)

$$-\frac{Gm\Delta m}{r^2}$$

- *net* pressure from the surrounding gas

$$[P(r) - P(r+\mathrm{d}r)]\,\mathrm{d}S$$

$\Rightarrow$ Acceleration

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\Delta m = -\frac{Gm\Delta m}{r^2} + [P(r) - P(r+\mathrm{d}r)]\,\mathrm{d}S$$

Equation of Motion

Using

$$P(r+\mathrm{d}r) = P(r) + \frac{\partial P}{\partial r}\,\mathrm{d}r \quad , \quad \Delta m = \rho\,\mathrm{d}r\,\mathrm{d}S$$

we obtain

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\Delta m = -\frac{Gm\,\Delta m}{r^2} - \frac{\partial P}{\partial r}\frac{\Delta m}{\rho}$$

or

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{Gm}{r^2} - \frac{1}{\rho}\frac{\partial P}{\partial r}$$

Using $\mathrm{d}r = \mathrm{d}m/(4\pi r^2 \rho)$ we can write the *Equation of Motion* as

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{Gm}{r^2} - 4\pi r^2 \frac{\partial P}{\partial m}$$

Hydrostatic Equilibrium

Neglecting acceleration we obtain the equation for hydrostatic equilibrium

radius coordinate: $$\frac{\partial P}{\partial r} = -\rho \frac{Gm}{r^2}$$

mass coordinate: $$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

NOTE

- pressure decreases outward
- pressure gradient vanishes at the center

Quiz

Show that the pressure gradient vanishes at the center.
Discuss with your neighbors. Really.

Central pressure of the star

Assume at surface $P(M) \approx 0$ we compute

$$P(0) = -\int_0^M \frac{Gm}{4\pi r^4}\,\mathrm{d}m > -\int_0^M \frac{Gm}{4\pi R^4}\mathrm{d}m = \frac{GM^2}{8\pi R^4}$$

Numerically...

$$P_\mathrm{c} > 4.4\times10^{18}\,\frac{\mathrm{dyn}}{\mathrm{cm}^2}\left(\frac{M}{\mathrm{M}_\odot}\right)\left(\frac{\mathrm{R}_\odot}{R}\right)^4$$

For the sun this is more than 450 million atmospheres.

Nuclear Reactions

Each species i is defined by its mass number A_i and charge number Z_i.

We assume that nuclear reactions

- conserve number of nucleons

$$\sum_{\text{in}} A_i = \sum_{\text{out}} A_i$$

- conserve total charge

$$\sum_{\text{in}} Z_i = \sum_{\text{out}} Z_i$$

Mass Fractions - Definitions

Assume species of partial density ρ_i, charge number Z_i, and mass number A_i.
We define

- *mass fraction*

$$X_i = \frac{\rho_i}{\rho}$$

- *number density*

$$n_i = \frac{\rho_i}{A_i m_\mathrm{H}}$$

- *mole fraction*

$$Y_i = \frac{\rho_i}{A_i \rho}$$

Note that instead of m_H the atomic mass unit u (1/12 the mass of the neutral $^{12}\mathrm{C}$ atom, $u = \frac{1}{12} m_{^{12}\mathrm{C}}$) should be used.

Mass Fractions

We obtain the relations

- $$n_i = \frac{\rho}{m_{\mathrm{H}}} \frac{X_i}{A_i}$$
- $$X_i = n_i \frac{A_i}{\rho} m_{\mathrm{H}}$$
- $$Y_i = \frac{X_i}{A_i}$$

Stellar Structure Equations

stationary terms time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{6}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \tag{7}$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{8}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \tag{9}$$

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) \tag{10}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Change of Composition: Mixing and Burning

The local composition, $\mathbf{X}(m,t)$, can change due to nuclear reactions and due to "*mixing*" processes inside the star.

$$\frac{\partial}{\partial t}X_i = f_{i,\mathrm{nuc}}\left(\rho, T, \mathbf{X}\right) + f_{i,\mathrm{mix}}\left(\rho, T, \mathbf{X}\right)$$

Often, this is approximated as a decoupled diffusive process

$$\frac{\partial}{\partial t}X_i = f_{i,\mathrm{nuc}}\left(\rho, T, \mathbf{X}\right) - \frac{\partial}{\partial m}\left(D_m \frac{\partial}{\partial m}X_i\right)$$

where the *mass diffusion coefficient*, D_m, is determined by the physical processes inside the stars. In radiative regions it is usually small, whereas it is large in *convective* regions. Convective regions evolve chemically homogeneously.

Nuclear Reactions

In a very general form nuclear reactions can be written as α_1 nuclei of species 1 plus α_2 nuclei of species 2 ... react to β_1 nuclei of species 1 plus β_2 nuclei of species 2 ... and reverse:

$$\alpha_1\mathbf{1}+\alpha_2\mathbf{2}+\ldots \rightleftharpoons \beta_1\mathbf{1}+\beta_2\mathbf{2}+\ldots$$

$Y_i = X_i/A_i$ is the mole fraction of nuclei i per mole nucleons.

The total rate of change of species i due to nuclear reactions can then be written as (for species **1**, **2**, ...)

$$\frac{\partial}{\partial t}Y_i = \sum_{\substack{\alpha_1,\alpha_2,\ldots\\ \beta_1,\beta_2,\ldots}} \lambda_{\alpha_1\mathbf{1}+\alpha_2\mathbf{2}+\ldots\rightarrow\beta_1\mathbf{1}+\beta_2\mathbf{2}+\ldots}\frac{\beta_i-\alpha_i}{\alpha_1!\alpha_2!\ldots}Y_1^{\alpha_1}Y_2^{\alpha_2}\ldots$$

Where the reaction rate $\lambda_{\ldots} \propto \rho^{-1+\alpha_1+\alpha_2+\ldots}$

Overview - Burning Phases in Stars

$20\,M_\odot$ star

Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	He	^{14}N	0.02	10^7	$4\,H \xrightarrow{CNO} {}^4He$
He	O, C	^{18}O, ^{22}Ne s-process	0.2	10^6	$3\,He^4 \rightarrow {}^{12}C$ $^{12}C(\alpha,\gamma)^{16}O$
C	Ne, Mg	Na	0.8	10^3	$^{12}C + {}^{12}C$
Ne	O, Mg	Al, P	1.5	3	$^{20}Ne(\gamma,\alpha)^{16}O$ $^{20}Ne(\alpha,\gamma)^{24}Mg$
O	Si, S	Cl, Ar, K, Ca	2.0	0.8	$^{16}O + {}^{16}O$
Si, S	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	$^{28}Si(\gamma,\alpha)\ldots$

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Summary

- Equation of Motion

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{Gm}{r^2} - 4\pi r^2 \frac{\partial P}{\partial m}$$

- hydrostatic equilibrium

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

- change of composition

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) = f_{i,\mathrm{nuc}}\left(\rho, T, \mathbf{X}\right) + f_{i,\mathrm{mix}}\left(\rho, T, \mathbf{X}\right)$$

- nuclear reactions

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}} \lambda_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \rightarrow \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots} \frac{\beta_i - \alpha_i}{\alpha_1! \alpha_2! \ldots} Y_1^{\alpha_1} Y_2^{\alpha_2} \ldots$$

Stellar Evolution Project

- Bill Paxton's **EZ Stellar Evolution** code
  `http://www.kitp.ucsb.edu/~paxton/EZ-intro.html`
- Uses Linux `gfortran`
- g95 FORTRAN compiler can be downloaded for most platforms.
  `http://www.g95.org`