Astrophysics I: Stars and Stellar Evolution

AST 4001

Alexander Heger[1,2,3]

[1] School of Physics and Astronomy
University of Minnesota

[2] Theoretical Astrophysics Group, T-6
Los Alamos National Laboratory

[3] Department of Astronomy and Astrophysics
University of California at Santa Cruz

Stars and Stellar Evolution, Fall 2008

Agenda

Overview

1. Recap
   - Web site access
   - Stellar Structure Equations
2. Stellar Structure Equations
   - Nuclear Reactions
   - Binding Energy and Energy Release Rate
   - Nuclear Burning Phases in Stars
3. Summary
   - Stellar Structure equations
   - Build Your Own Star

Contact

- **Location & Dates:**
  Physics 236A, MTWTh 10:10-11:00 AM
- **Office hours:**
  Wednesdays, 13:00-14:30, 342F Tate
- **email:**
  I cannot guarantee that I will receive all emails due to SPAM filters. On class days I will try to reply to email within 24 h.
- **Web site:**
  `http://stellarevolution.org/AST-4001`
  I will post notes, updates, problem sets, etc.
- **Google course calendar** (on Web site):
  `o86pe6r5paic30h4qv6acm9ej0%40group.calendar.google.com`

Web site access

- user name: **Ast-4001**
- password: **&32yˆnbY**

Summary

- Equation of Motion

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{Gm}{r^2} - 4\pi r^2 \frac{\partial P}{\partial m}$$

- hydrostatic equilibrium

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

- change of composition

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) = f_{i,\mathrm{nuc}}\left(\rho, T, \mathbf{X}\right) + f_{i,\mathrm{mix}}\left(\rho, T, \mathbf{X}\right)$$

- nuclear reactions

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}} \lambda_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \rightarrow \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots} \frac{\beta_i - \alpha_i}{\alpha_1 ! \alpha_2 ! \ldots} Y_1^{\alpha_1} Y_2^{\alpha_2} \ldots$$

Mass Fractions - Definitions

Assume species of partial density ρ_i, charge number Z_i, and mass number A_i.
We define

- *mass fraction*

$$X_i = \frac{\rho_i}{\rho}$$

- *number density*

$$n_i = \frac{\rho_i}{A_i \mathrm{u}}$$

- *mole fraction*

$$Y_i = \frac{\rho_i}{A_i \rho}$$

Note that instead of m_{H} the atomic mass unit u (1/12 the mass of the neutral $^{12}\mathrm{C}$ atom, $\mathrm{u} = \frac{1}{12} m_{^{12}\mathrm{C}}$) should be used.

Change of Composition: Mixing and Burning

The local composition, $\mathbf{X}(m, t)$, can change due to nuclear reactions and due to "*mixing*" processes inside the star.

$$\frac{\partial}{\partial t} X_i = f_{i,\mathrm{nuc}}\left(\rho, T, \mathbf{X}\right) + f_{i,\mathrm{mix}}\left(\rho, T, \mathbf{X}\right)$$

Often, this is approximated as a decoupled diffusive process

$$f_{i,\mathrm{mix}}\left(\rho, T, \mathbf{X}\right) = -\frac{\partial}{\partial m}\left(D_m \frac{\partial}{\partial m} X_i\right)$$

where the *mass diffusion coefficient*, D_m, is determined by the physical processes inside the stars. In radiative regions it is usually small, whereas it is large in *convective* regions. Convective regions evolve chemically homogeneously.

Overview - Burning Phases in Stars

$20\,M_\odot$ star

Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	He	^{14}N	0.02	10^7	4 H $\xrightarrow{\text{CNO}}$ ^{4}He
He	O, C	^{18}O, ^{22}Ne s-process	0.2	10^6	3 He^4 → ^{12}C ^{12}C(α,γ)^{16}O
C	Ne, Mg	Na	0.8	10^3	^{12}C + ^{12}C
Ne	O, Mg	Al, P	1.5	3	^{20}Ne(γ,α)^{16}O ^{20}Ne(α,γ)^{24}Mg
O	Si, S	Cl, Ar, K, Ca	2.0	0.8	^{16}O + ^{16}O
Si, S	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	^{28}Si(γ,α)…

Overview

Stellar Structure Equations - Nuclear Burning

stationary terms　　time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad (1)$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \quad (2)$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \quad (3)$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \quad (4)$$

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) \quad (5)$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Nuclear Reactions

In a very general form nuclear reactions can be written as α_1 nuclei of species 1 plus α_2 nuclei of species 2 ... react to β_1 nuclei of species 1 plus β_2 nuclei of species 2 ... and reverse:

$$\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \rightleftharpoons \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots$$

$Y_i = X_i / A_i$ is the mole fraction of nuclei i per mole nucleons.

The total rate of change of species i due to nuclear reactions can then be written as (for species **1**, **2**, ...)

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}} \lambda_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \rightarrow \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots} \frac{\beta_i - \alpha_i}{\alpha_1 ! \alpha_2 ! \ldots} Y_1^{\alpha_1} Y_2^{\alpha_2} \ldots$$

Where the reaction rate $\lambda_{\ldots} \propto \rho^{-1+\alpha_1+\alpha_2+\ldots}$

Nuclear Reactions - Example 1 - an (a,p) reaction

The binary reaction

$$^{9}\mathrm{C} + {}^{4}\mathrm{He} \longmapsto {}^{12}\mathrm{N} + {}^{1}\mathrm{H}$$

with **1** $= {}^{9}\mathrm{C}$, **2** $= {}^{4}\mathrm{He}$, **3** $= {}^{12}\mathrm{N}$, and **4** $= {}^{1}\mathrm{H}$ gives system

$$\frac{\partial}{\partial t} Y_{^{9}\mathrm{C}} = -\lambda_{^{9}\mathrm{C}+^{4}\mathrm{He}\longmapsto^{12}\mathrm{N}+^{1}\mathrm{H}} Y_{^{9}\mathrm{C}} Y_{^{4}\mathrm{He}}$$

$$\frac{\partial}{\partial t} Y_{^{4}\mathrm{He}} = -\lambda_{^{9}\mathrm{C}+^{4}\mathrm{He}\longmapsto^{12}\mathrm{N}+^{1}\mathrm{H}} Y_{^{9}\mathrm{C}} Y_{^{4}\mathrm{He}}$$

$$\frac{\partial}{\partial t} Y_{^{12}\mathrm{N}} = \lambda_{^{9}\mathrm{C}+^{4}\mathrm{He}\longmapsto^{12}\mathrm{N}+^{1}\mathrm{H}} Y_{^{9}\mathrm{C}} Y_{^{4}\mathrm{He}}$$

$$\frac{\partial}{\partial t} Y_{^{1}\mathrm{H}} = \lambda_{^{9}\mathrm{C}+^{4}\mathrm{He}\longmapsto^{12}\mathrm{N}+^{1}\mathrm{H}} Y_{^{9}\mathrm{C}} Y_{^{4}\mathrm{He}}$$

Nuclear Reactions - Example 2 - 3α reaction

The reaction

$$3\,{}^{4}\mathrm{He} \longmapsto {}^{12}\mathrm{C}$$

with $\mathbf{1} = {}^{4}\mathrm{He}$, $\mathbf{2} = {}^{12}\mathrm{C}$, gives system

$$\frac{\partial}{\partial t} Y_{^4\mathrm{He}} = -\frac{1}{2}\lambda_{3\,{}^4\mathrm{He}\longmapsto{}^{12}\mathrm{C}}\, Y^3_{^4\mathrm{He}}$$

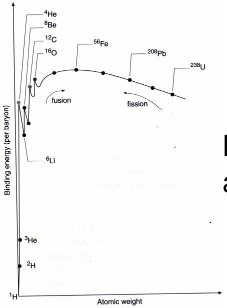
$$\frac{\partial}{\partial t} Y_{^{12}\mathrm{C}} = \frac{1}{6}\lambda_{3\,{}^4\mathrm{He}\longmapsto{}^{12}\mathrm{C}}\, Y^3_{^4\mathrm{He}}$$

Mass Excess

The *mass excess* $\Delta\mathcal{M}_i$ of a nucleus (isotope), i, is given by the rest mass of the neutral atom minus $A_i\mathrm{u}$ ($\mathrm{u} = \frac{1}{12} m_{^{12}\mathrm{C}}$, mass of neutral $^{12}\mathrm{C}$ atom).

The energy release of a nuclear reaction is then given by

$$Q_{\ldots} = c^2 \times (\text{(total mass excess of reactants)} - \text{(total mass excess of products)})$$



Binding energy as a function of atomic weight

Energy Release by Nuclear Burning

For a reaction of type

$$\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \rightleftharpoons \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots$$

the energy release is hence given by

$$\Delta \mathcal{Q} = c^2 \left(\sum_i \alpha_i \Delta \mathcal{M}_i - \sum_i \beta_i \Delta \mathcal{M}_i \right)$$

and the energy release *rate* is given by

$$\varepsilon^{\mathrm{nuc}}_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \to \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots} =$$

$$c^2 \sum_{\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}} \lambda_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \to \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots} \frac{\beta_i - \alpha_i}{\alpha_1 ! \alpha_2 ! \ldots} Y_1^{\alpha_1} Y_2^{\alpha_2} \ldots \times \left(\sum_i \alpha_i \Delta \mathcal{M}_i - \sum_i \beta_i \Delta \mathcal{M}_i \right)$$

Energy Release by Nuclear Burning (II)

The total energy release rate due to burning by all reactions is given by

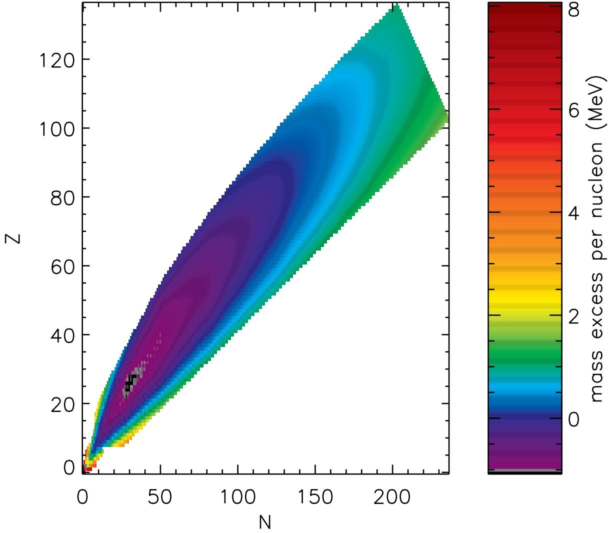
$$\varepsilon_{\mathrm{nuc}} = \sum_{\mathrm{i}\in\mathrm{reactions}} \varepsilon_i^{\mathrm{nuc}} = -c^2 \frac{\partial}{\partial t} \sum_i Y_i \Delta\mathcal{M}_i$$

$$\varepsilon_{\mathrm{nuc}} = -c^2 \sum_i \Delta\mathcal{M}_i \frac{\partial}{\partial t} Y_i \ .$$

NOTE

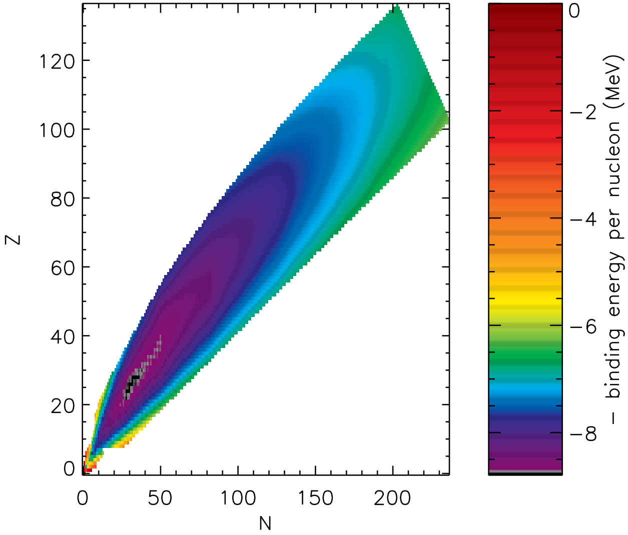
The Binding energy of a nucleus, i.e., the energy needed to separate a nucleus into its constituents, the nucleons, is different from the mass excess. This is because $^{12}\mathrm{C}$ consists of equal number of protons and neutrons, while most nuclei do not. Protons and neutrons have different mass excess.

Nuclear Mass Excess



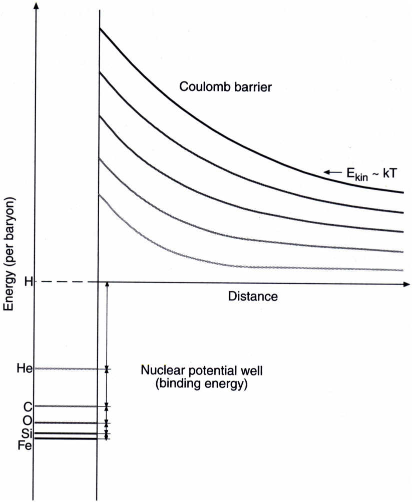
Mass of nuclei relative to $\frac{1}{12} \times {}^{12}\text{C} \times$ (number of Nucleons = mass number A)

Nuclear Binding Energy



Energy required to separate nuclei into free nucleons.

Nuclear Reaction Rates



Reduced mass in frame of interaction

$$m_{\mathrm{red}} = \frac{m_1 m_2}{m_1 + m_2}$$

Separation distance

$$d = \frac{Z_1 Z_2 e^2}{\frac{1}{2} m_{\mathrm{red}} v^2}$$

Barrier penetration probability

$$\propto \exp\left\{-4\pi^2 \frac{Z_1 Z_2 e^2}{hv}\right\}$$

The Gamow Window

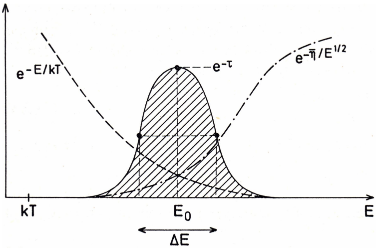
Assuming energy distribution of gas particles is given by Maxwell Distribution

$$n(p)\mathrm{d}p = \frac{4\pi p^2}{\sqrt{2\pi mkT}^3} e^{-\frac{p^2}{2mkT}}$$

The probability of a particle being in velocity bin between v and $v + \mathrm{d}v$ is hence

$$\propto e^{-\frac{p^2}{2mkT}} = \propto e^{-\frac{mv^2}{2kT}}$$

The Gamow Window (2)



The product penetration probability and velocity distribution is therefore proportional to

$$e^{-4\pi^2 \frac{Z_1 Z_2 e^2}{hv}} e^{\frac{-mv^2}{2kT}}$$

The Gamow Window (3)

The probability has a maximum for a velocity of

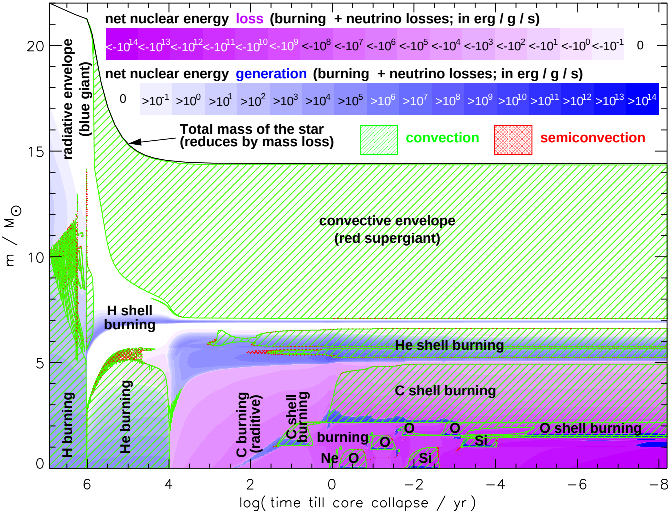
$$v = \left(-4\pi^2 \frac{Z_1 Z_2 e^2 kT}{hm}\right)^{1/3}$$

Integrating over the entire probability distribution one finds that the resulting reaction rate is proportional to

$$\langle \sigma v \rangle \propto (kT)^{-2/3}\, e^{-\frac{3}{2}\left(\frac{4\pi^2 Z_1 Z_2 e^2}{h}\right)^{2/3}\left(\frac{m}{kT}\right)^{1/3}}$$

Generally, $\langle \sigma v \rangle$ can be approximated for a small range of temperatures relevant for a reaction as a power law, $\varepsilon_{\mathrm{nuc}} \propto T^n$.

Overview - Burning Phases in the Stellar Interior



Stellar Evolution as a Function of Mass

(Stellar Evolution as a Function of Mass)

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Summary

- nuclear reactions

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1,\alpha_2,\dots \\ \beta_1,\beta_2,\dots}} \lambda_{\alpha_1 \mathbf{1}+\alpha_2 \mathbf{2}+\ldots \rightarrow \beta_1 \mathbf{1}+\beta_2 \mathbf{2}+\ldots} \frac{\beta_i - \alpha_i}{\alpha_1!\alpha_2!\ldots} Y_1^{\alpha_1} Y_2^{\alpha_2} \ldots$$

- mass excess

$$\Delta\mathcal{M}_i = m_i - A_i \mathrm{u}$$

- nuclear energy release

$$\varepsilon_{\mathrm{nuc}} = -c^2 \sum_i \Delta\mathcal{M}_i \frac{\partial}{\partial t} Y_i$$

- reaction rate

$$\langle \sigma v \rangle \propto (kT)^{-2/3} \, e^{-\frac{3}{2}\left(\frac{4\pi^2 Z_1 Z_2 e^2}{h}\right)^{2/3}\left(\frac{m}{kT}\right)^{1/3}}$$

Stellar Evolution Project

- Bill Paxton's **EZ Stellar Evolution** code
  `http://www.kitp.ucsb.edu/~paxton/EZ-intro.html`
- Uses Linux `gfortran`
- g95 FORTRAN compiler can be downloaded for most platforms.
  `http://www.g95.org`