Astrophysics I: Stars and Stellar Evolution

AST 4001

Alexander Heger[1,2,3]

[1] School of Physics and Astronomy
University of Minnesota

[2] Theoretical Astrophysics Group, T-6
Los Alamos National Laboratory

[3] Department of Astronomy and Astrophysics
University of California at Santa Cruz

Stars and Stellar Evolution, Fall 2008

Agenda

Overview

Contact

- **Location & Dates:**
  Physics 236A, MTWTh 10:10-11:00 AM
- **Office hours:**
  Wednesdays, 13:00-14:30, 342F Tate
- **email:**
  I cannot guarantee that I will receive all emails due to SPAM filters. On class days I will try to reply to email within 24 h.
- **Web site:**
  `http://stellarevolution.org/AST-4001`
  I will post notes, updates, problem sets, etc.
- **Google course calendar** (on Web site):
  `o86pe6r5paic30h4qv6acm9ej0%40group.calendar.google.com`

Web site access

- user name: **Ast-4001**
- password: **&32yˆnbY**

Summary (I)

- nuclear reactions

$$\frac{\partial}{\partial t}Y_i = \sum_{\substack{\alpha_1,\,\alpha_2,\,\ldots\\ \beta_1,\,\beta_2,\,\ldots}} \lambda_{\alpha_1\mathbf{1}+\alpha_2\mathbf{2}+\ldots\to\beta_1\mathbf{1}+\beta_2\mathbf{2}+\ldots}\frac{\beta_i-\alpha_i}{\alpha_1!\alpha_2!\ldots}Y_1^{\alpha_1}Y_2^{\alpha_2}\ldots$$

- mass excess

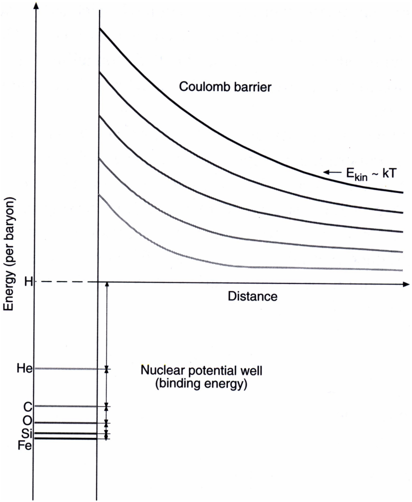
$$\Delta\mathcal{M}_i = m_i - A_i\mathrm{u}$$

- nuclear energy release

$$\varepsilon_{\mathrm{nuc}} = -c^2\sum_i \Delta\mathcal{M}_i\frac{\partial}{\partial t}Y_i$$

- a cool page with nuclear data
  `http://wwwndc.jaea.go.jp/CN04/index.html`

Nuclear Reaction Rates



Reduced mass in frame of interaction

$$m_{\text{red}} = \frac{m_1 m_2}{m_1 + m_2}$$

Separation distance

$$d = \frac{Z_1 Z_2 e^2}{\frac{1}{2} m_{\text{red}} v^2}$$

Barrier penetration probability

$$\propto \exp\left\{-4\pi^2 \frac{Z_1 Z_2 e^2}{hv}\right\}$$

The Gamow Window

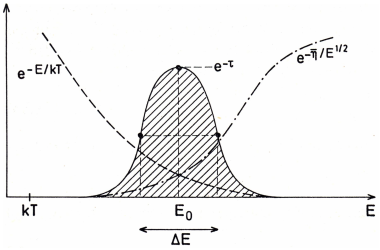
Assuming energy distribution of gas particles is given by Maxwell Distribution

$$n(p)\mathrm{d}p = \frac{4\pi p^2}{\sqrt{2\pi mkT}^3} e^{-\frac{p^2}{2mkT}}$$

The probability of a particle being in velocity bin between v and $v + \mathrm{d}v$ is hence

$$\propto e^{-\frac{p^2}{2mkT}} = e^{-\frac{mv^2}{2kT}}$$

The Gamow Window (2)



The product penetration probability and velocity distribution is therefore proportional to

$$e^{-4\pi^2 \frac{Z_1 Z_2 e^2}{h v}} \, e^{\frac{-mv^2}{2kT}}$$

The Gamow Window (3)

The probability has a maximum for a velocity of

$$v = \left(-4\pi^2 \frac{Z_1 Z_2 e^2 kT}{hm}\right)^{1/3}$$

Integrating over the entire probability distribution one finds that the resulting reaction rate is proportional to

$$\langle \sigma v \rangle \propto (kT)^{-2/3}\, e^{-\frac{3}{2}\left(\frac{4\pi^2 Z_1 Z_2 e^2}{h}\right)^{2/3}\left(\frac{m}{kT}\right)^{1/3}}$$

Generally, $\langle \sigma v \rangle$ can be approximated for a small range of temperatures relevant for a reaction as a power law, $\varepsilon_{\mathrm{nuc}} \propto T^n$.

Astrophysical S-factor

Note that the Gamov peak has width of

$$\Delta = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} \ll E_0$$

Generally one computes the reaction rate from

$$\langle \sigma v \rangle = \int_0^\infty \sigma(E)\, v\, f(E)\, \mathrm{d}E$$

$$f(E) = \frac{2}{\sqrt{\pi}} \frac{E^{1/2}}{(kT)^{3/2}} e^{-E/kT}\, \mathrm{d}E$$

$$\sigma(E) = \mathbf{S} E^{-1} e^{-2\pi\eta}\,, \quad \eta = \left(\frac{m}{2}\right)^{1/2} \frac{2\pi Z_1 Z_2 e^2}{h E^{1/2}}$$

defines *astrophysical S-factor*.

Astrophysical S-factor

Evaluating of the Gamov Peak assuming a Gaussian, for non-resonant reactions one approximates:

$$\langle \sigma v \rangle = S_{\mathrm{eff}} \left(\frac{1}{3} \right)^{1/2} \left(\frac{16\pi^2}{m} \frac{Z_1 Z_2 e^2}{h} \right)^{1/3} e^{-\frac{3}{2} \left(\frac{4\pi^2 Z_1 Z_2 e^2}{h} \right)^{2/3} \left(\frac{m}{kT} \right)^{1/3}}$$

Where $S_{\mathrm{eff}} \approx S(E_0)$ and E_0 is the energy of the Gamov peak. More accurately, one has with $S(E) = S(0) + S'(0)E + \frac{1}{2}S''(0)E^2$:

$$S_{\mathrm{eff}}(E_0) = S(0) \left[1 + \frac{5}{12\tau} + \frac{S'(0)}{S(0)} \left(E_0 + \frac{35}{36} kT \right) + \frac{1}{2} \frac{S''(0)}{S(0)} \left(E_0^2 + \frac{89}{36} E_0 kT \right) \right]$$

$$\tau = 3E_0/kT = 3 \left(2\pi^2 \left(\frac{m}{2kT} \right)^{1/2} \frac{Z_1 Z_2 e^2}{h} \right)^{2/3}$$

Notes on Reactions

- For *resonant* nuclear reactions very different temperature sensitivities result.
- Calculating S(E) accurately from first principles is very difficult and possible with any accuracy only in very limited cases. Current state of the art is to use statistical models for nuclei.
- In general $S(E)$ is determined from experiment.
- Measurement of $S(E)$ for astrophysically relevant energies is difficult. Usually one measures at high E and extrapolates to $E \to 0$ (i.e., to approximately $S(E = 0)$).

Summary (II)

- reaction rate

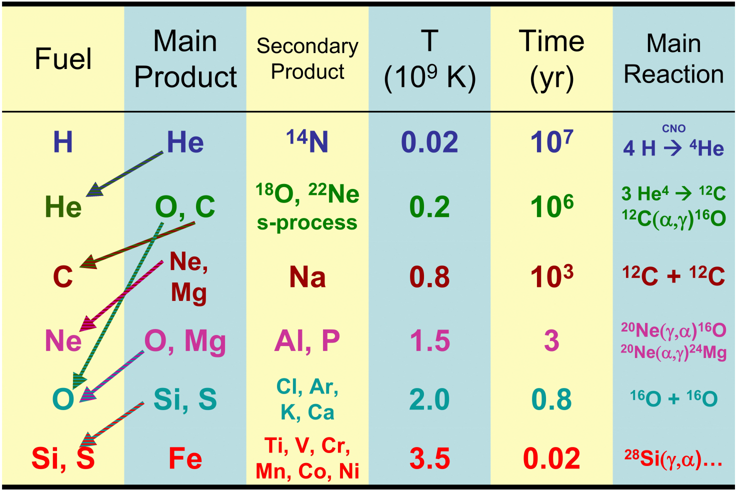
$$\langle \sigma v \rangle \propto (kT)^{-2/3}\, e^{-\frac{3}{2}\left(\frac{4\pi^2 Z_1 Z_2 e^2}{h}\right)^{2/3}\left(\frac{m}{kT}\right)^{1/3}}$$

- relation between $\langle \sigma v \rangle$ and λ

$$\lambda \propto \langle \sigma v \rangle\, \rho^m\,, \quad m = \sum \alpha_i - 1$$

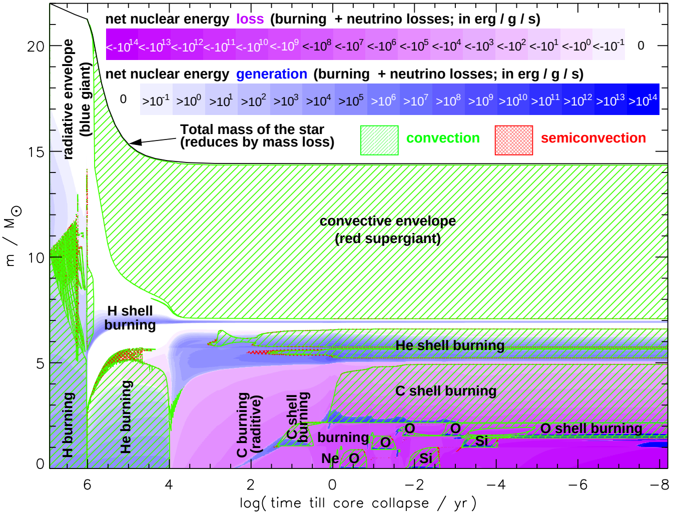
Overview - Burning Phases in Stars

$20\,M_\odot$ star



Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	**He**	**^{14}N**	**0.02**	**10^7**	**$4\,H \xrightarrow{CNO} {}^4He$**
He	**O, C**	**^{18}O, ^{22}Ne s-process**	**0.2**	**10^6**	**$3\,He^4 \rightarrow {}^{12}C$ $^{12}C(\alpha,\gamma)^{16}O$**
C	**Ne, Mg**	**Na**	**0.8**	**10^3**	**$^{12}C + {}^{12}C$**
Ne	**O, Mg**	**Al, P**	**1.5**	**3**	**$^{20}Ne(\gamma,\alpha)^{16}O$ $^{20}Ne(\alpha,\gamma)^{24}Mg$**
O	**Si, S**	**Cl, Ar, K, Ca**	**2.0**	**0.8**	**$^{16}O + {}^{16}O$**
Si, S	**Fe**	**Ti, V, Cr, Mn, Co, Ni**	**3.5**	**0.02**	**$^{28}Si(\gamma,\alpha)\ldots$**

Overview - Burning Phases in the Stellar Interior



Stellar Evolution as a Function of Mass

(Stellar Evolution as a Function of Mass)

http://stellarevolution.org/movie/sfw.gif

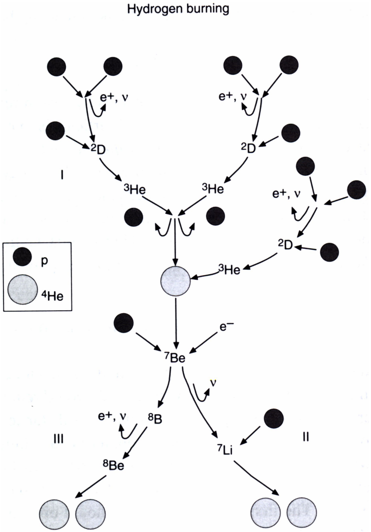
Overview

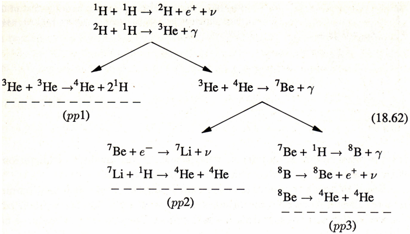
1. Recap
   - Nuclear Reactions and Energy Release
   - Nuclear Reactions Rates
   - Nuclear Burning Phases in Stars
2. Hydrogen Burning in Stars
   - pp-chain
   - CNO cycle
   - Competition of burning modes
3. Summary
   - Hydrogen Burning
   - Build Your Own Star

Hydrogen Burning - the two modes

- Two basic modes of hydrogen burning are distinguished
- The pp-chain in low-mass stars
- The Carbon-Nitrogen-Oxygen (CNO) cycle in high-mass stars

Hydrogen Burning - pp chains





$$^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu$$
$$^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma$$

$$^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H} \quad (pp1)$$

$$^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma \qquad (18.62)$$

$$^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu$$
$$^7\mathrm{Li} + {}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} \quad (pp2)$$

$$^7\mathrm{Be} + {}^1\mathrm{H} \rightarrow {}^8\mathrm{B} + \gamma$$
$$^8\mathrm{B} \rightarrow {}^8\mathrm{Be} + e^+ + \nu$$
$$^8\mathrm{Be} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} \quad (pp3)$$

Energy release:

$Q(pp1) = 26.20\,MeV$

$Q(pp2) = 25.67\,MeV$

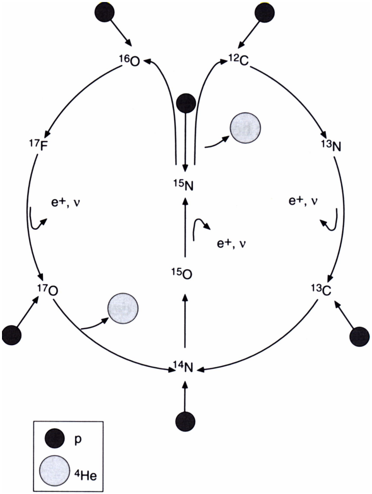
$Q(pp3) = 19.20\,MeV$

Reaction rate: $\langle \sigma v \rangle \propto T^4$

Notes on pp hydrogen burning

- All four chains fuse 4 protons to one $^{4}\mathrm{He}$ – and therefore have the same difference in mass excess. They have the same energy supply.
- Different Q-values (amount of energy release) due to different amounts of energy being *carried away by neutrinos*.
- With increasing temperature the dominant burning switches from pp1 to pp2 to pp3 chains.

Hydrogen Burning - CNO bi-cycle

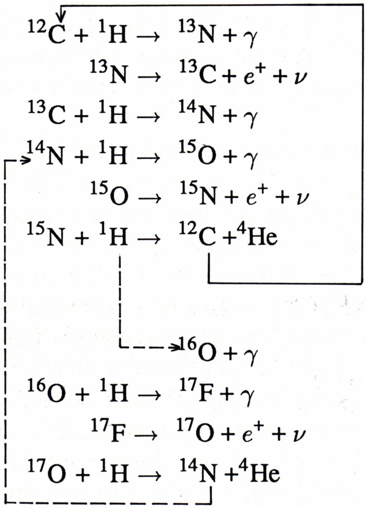


Energy release:
$Q(CNO) = 24.97 MeV$

Reaction rate: $\langle \sigma v \rangle \propto T^{16}$

Branching:
CNO-1 : CNO-2 $\sim$ 10,000 : 1

Hydrogen Burning - CNO bi-cycle



$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$$
$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu$$
$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$$
$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$$
$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu$$
$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$$
$$\rightarrow {}^{16}O + \gamma$$
$$^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$$
$$^{17}F \rightarrow {}^{17}O + e^{+} + \nu$$
$$^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$$

Energy release:
$Q(CNO) = 24.97 MeV$

Reaction rate: $\langle \sigma v \rangle \propto T^{16}$

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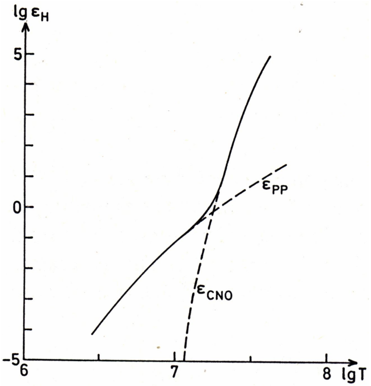
CNO-1 : CNO-2 $\sim$ 10,000 : 1

Hydrogen Burning - CNO bi-cycle

- Usually the beta-decays are fast compared to the capture reactions, (p,γ).
- $^{14}\mathrm{O}$: $\tau_{1/2} = 70$ sec
  $^{15}\mathrm{O}$: $\tau_{1/2} = 122$ sec
  $^{13}\mathrm{N}$: $\tau_{1/2} = 10\,\mathrm{min}$
  $^{17}\mathrm{F}$: $\tau_{1/2} = 64$ sec
  $^{18}\mathrm{O}$: $\tau_{1/2} = 110\,\mathrm{min}$
- $^{14}\mathrm{N}(\mathrm{p},\gamma)^{15}\mathrm{O}$ usually is the slowest “bottleneck” reaction.
- CNO cycle burning converts most CNO isotopes into $^{14}\mathrm{N}$.

Competition of Hydrogen-Burning Modes



Transition from pp-chains
in low-mass stars (low *T*)
to CNO chains
in high-mass stars (high *T*)

Was that all of hydrogen burning...?

Future questions on hydrogen burning

- What happens at the first stars? (no initial CNO)
- What happens at high temperatures?

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Summary

- hydrogen burning can proceeds in different modes. Which mode dominates depending on temperature and the mass of the star.
- We destinguish
    - pp-chain(s)
    - CNO (bi-)cycle

Stellar Evolution Project

- Bill Paxton's **EZ Stellar Evolution** code
  `http://www.kitp.ucsb.edu/~paxton/EZ-intro.html`
- Uses Linux `gfortran`
- g95 FORTRAN compiler can be downloaded for most platforms.
  `http://www.g95.org`