Astrophysics I: Stars and Stellar Evolution
AST 4001

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Stars and Stellar Evolution, Fall 2008

Announcement

William I. Fine
Theoretical Physics Institute
University of Minnesota

The *3rd* Annual Irving and Edythe
MISEL *FAMILY* LECTURE

Professor Jim Peebles
Princeton University
Albert Einstein Professor of Science Emeritus
Finding the Big Bang

Public Lecture
Tuesday, September 23, 2008
7:00pm
Van Vleck Auditorium
Room 150, Tate Lab of Physics

Physics and Astronomy Colloquium
The Cosmological Tests
Wednesday, September 24, 2008
3:35pm
Room 131 Tate Lab of Physics

For more information visit: www.ftpi.umn.edu/misel/

Announcement

Burning Neutron Stars

Alexander Heger

Friday, September 26, 15:00, PHYS 435

The probably by far most common thermonuclear explosion to occur in nature is the explosion of a thin layer of material, about the height of the physics building, that has accumulated on the surface of a neutron star, about the size of Minneapolis, in a binary star system - Type I X-ray bursts. I show theoretical models for such outbursts, their very specific mode of nuclear burning unheard of in any other stellar system, as well as their much bigger cousins, the superbursts. I will discuss our current difficulty in understanding how those are made, and possible solutions.

Agenda

Overview

1. Recap
   - Summary
   - Adiabatic Index and Exponent
   - Saha Equation
2. EOS & Opacity
   - Equation of State from Pairs and Dissociation
   - Energy Transport and Opacity
3. Summary
   - Summary
   - Build Your Own Star

Astrophysical Constants in cgs

Physical and Astronomical Constants

CODATA Internationally recommended values of the Fundamental Physical Constants.
The most recent values can be found at `http://physics.nist.gov/cuu/Constants`.

Solar mass	$M_\odot$	=	$1.989 \cdot 10^{33}$ gm
Solar radius	$R_\odot$	=	$6.955 \cdot 10^{10}$ cm
Solar effective temperature	$T_{\mathrm{eff},\odot}$	=	5780 K
Solar surface gravity	$g_{s,\odot}$	=	$2.744 \cdot 10^4$ cm/sec^2
Solar luminosity	$L_\odot$	=	$3.846 \cdot 10^{33}$ erg/sec
Solar absolute bol. mag.	$M_{b,\odot}$	=	+4.77
Velocity of light in vacuo	c	=	$2.99792458 \cdot 10^{10}$ cm/sec
Constant of gravitation	G	=	$6.6742 \cdot 10^{-8}$ cm^3/(gm s^2)
Boltzmann constant	k	=	$1.3807 \cdot 10^{-16}$ erg/K
Avogadro's number	N_0	=	$6.022 \cdot 10^{23}$ mole^{-1}
Atomic mass unit	1 AMU	=	$1/N_0 = H$
		=	$1.66054 \cdot 10^{-24}$ gm = 931.5 MeV
Gas constant	$\mathcal{R}$	=	$8.314 \cdot 10^7$ erg/K/mole
Planck's constant	h	=	$6.626 \cdot 10^{-27}$ erg sec
	$\hbar = h/2\pi$	=	$1.0546 \cdot 10^{-27}$ erg sec
Electronic charge	e	=	$4.803 \cdot 10^{-10}$ e.s.u.
		=	$1.602 \cdot 10^{-19}$ C
Fine structure constant	$e^2/\hbar c$	=	1/137.036
Stefan-Boltzmann constant	σ	=	$5.670 \cdot 10^{-5}$ erg/(cm^2 K^4 sec)
Radiation pressure constant	$a = 4\sigma/c$	=	$7.566 \cdot 10^{-15}$ erg/(cm^3 K^4)
Electron rest mass	m_e	=	$9.109 \cdot 10^{-28}$ gm = 0.5110 MeV
Mass ratio proton/electron	m_p/m_e	=	1836.2
Mass of hydrogen atom	H^1	=	$1.6734 \cdot 10^{-24}$ gm
		=	1.0081 AMU
Classical electron radius	$e^2/m_e c^2$	=	$2.818 \cdot 10^{-13}$ cm
Compton wavelength of electron	$\lambda_C = \hbar/m_e c$	=	$3.8616 \cdot 10^{-11}$ cm
Thomson scattering cross section	σ_0	=	$(8\pi/3)\left(e^2/m_e c^2\right)^2$
		=	$0.6652 \cdot 10^{-24}$ cm^2
Electron volt	1 eV	=	$1.602 \cdot 10^{-12}$ erg = 11604 K

Summary of Stellar Gas

- non-relativistic gas

$$u_{\mathrm{gas}} = \frac{3}{2}\frac{P_{\mathrm{gas}}}{\rho}$$

- relativistic gas (ions *or* photons)

$$u_{\mathrm{rad}} = 3\frac{P_{\mathrm{rad}}}{\rho}$$

- adiabatic index

$$\gamma_{\mathrm{ad}} = \frac{\mathrm{d}\ \ln P}{\mathrm{d}\ \ln \rho} = \frac{\phi + 1}{\phi}$$

Adiabatic Index and Exponent

- We define the adiabatic exponent, γ_{ad} and the adiabatic index, ϕ by

$$\gamma_{\mathrm{ad}} = \frac{\mathrm{d}\ \ln P}{\mathrm{d}\ \ln \rho} = \frac{\phi + 1}{\phi}$$

- hence we have a relation between gas pressure and density

$$P = K_{\mathrm{ad}} \rho^{\gamma_{\mathrm{ad}}} \propto \rho^{\gamma_{\mathrm{ad}}}$$

Saha Equation & Ionization

For a simple gas with one ionization stage we have

- *degree of ionization*

$$x = \frac{n_+}{n_0 + n_+}$$

- the densities are related by the Saha Equation

$$\frac{n_+ \, n_{\mathrm{e}^-}}{n_0} = \frac{g}{h^3}(2\pi m_\mathrm{e} k_\mathrm{B} T)^{3/2} e^{-\chi/k_\mathrm{B} T}$$

- the pressure is then

$$P = (1+x)(n_0 + n_+)k_\mathrm{B} T = (1+x)\mathcal{R}\rho T$$

- the resulting adiabatic index is

$$\gamma_\mathrm{ad} = \frac{5 + \left(\frac{5}{2} + \frac{\chi}{k_\mathrm{B} T}\right)^2 x(1-x)}{3 + \left[\frac{3}{2} + \left(\frac{3}{2} + \frac{\chi}{k_\mathrm{B} T}\right)^2\right] x(1-x)}$$

Quiz

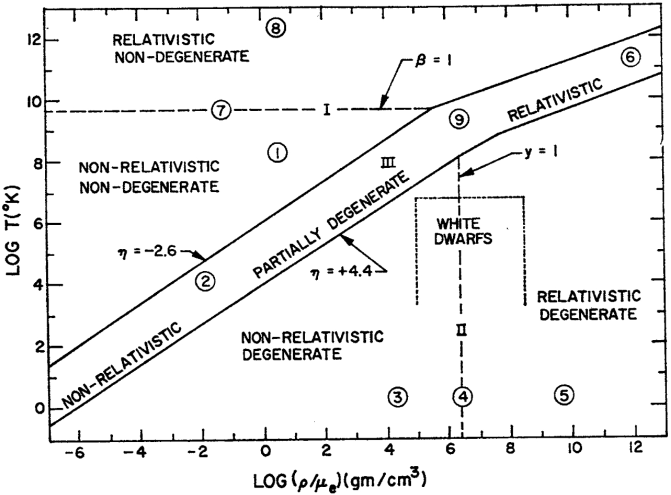
1. For a fully ionized gas, on what properties of the gas depends the constant K_{ad} in

$$P = K_{\mathrm{ad}} \rho^{\gamma_{\mathrm{ad}}} \quad ?$$

2. How does K_{ad} change during ionization?
Compare the values before and after ionization.

Overview

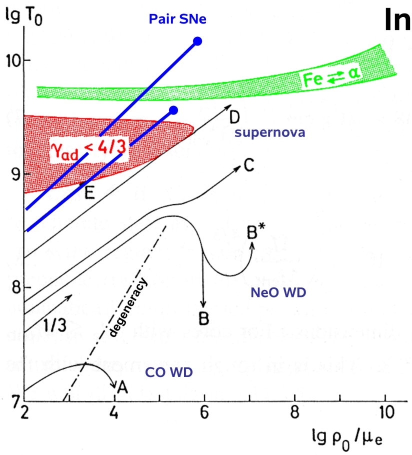
Electron Equation of State Regimes



Electron-Positron Pair Production

- At $T \gtrsim 1 \times 10^9$ K photon can produce electron-positron pairs, from the highest energy photons of the Planck spectrum, $h\nu > 2 m_{\mathrm{e}} c^2$:
$$e^+ + e^- \rightleftharpoons \gamma$$
- This converts radiation energy into rest mass of pairs
- hence compression increases pressure less
- $\Rightarrow$ adiabatic index γ_{ad} lower
- possible instability of star ($\gamma_{\mathrm{ad}} < \frac{4}{3}$)
"pair instability supernova"
($\gamma_{\mathrm{ad}} \gtrsim \frac{4}{3}$ is needed for stability of stars, as we shall see later)

Electron-Positron Pair Production and Iron Dissociation



Kippenhahn & Weigert (1990)

Instability Regimes

adiabatic index < 4/3

Compression does not result in sufficient increase in pressure (gradient) to balance higher gravity at lower radius

e^+/e^--Pair Instability

Internal gas energy is converted into e^+/e^- rest mass (hard photons from tail of Planck spectrum)

Photo disintegration

Internal gas energy is used to unbind heavy nuclei into alpha particles and at higher temperature those into free nucleons

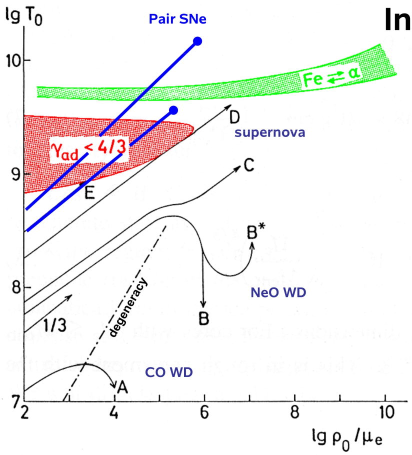
Iron Photo-Dissociation

- At very high temperatures in the stellar core, typically during the last stages of *massive* (and very massive) stars, including collapse of the iron core, iron can be dissociated, typically above $T > 7 \times 10^9$ K:

$$^{56}\mathrm{Fe} + \gamma \rightleftharpoons 13\,^{4}\mathrm{He} + 4\,\mathrm{n}$$

- This takes 100 MeV
- $\Rightarrow$ gas energy is used to unbind nucleus
- takes (about) as much energy as was released before to burn ^{4}He to ^{56}Fe
- $\Rightarrow$ γ_{ad} drops
- $\Rightarrow$ possible instability of star (collapse)

Electron-Positron Pair Production and Iron Dissociation



Kippenhahn & Weigert (1990)

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e^+/e^--Pair Instability

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Photo disintegration

Internal gas energy is used to unbind heavy nuclei into alpha particles and at higher temperature those into free nucleons

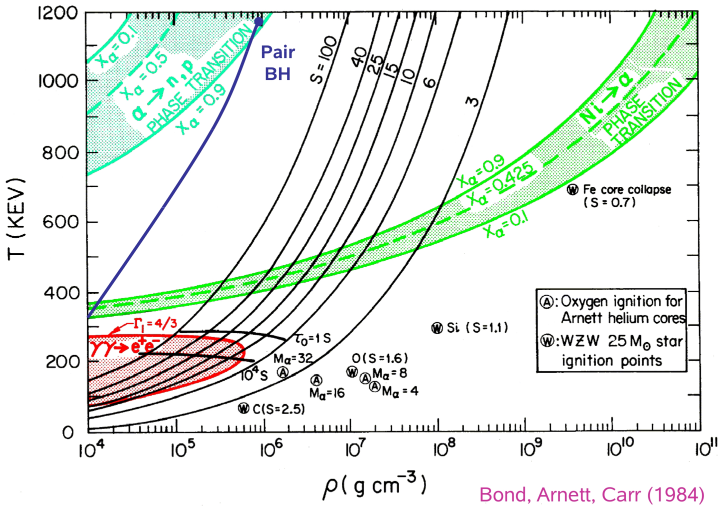
Helium Photo-Dissociation

- At even higher temperatures helium can be dissociated, typically above $T \gtrsim 10^{10}\,\mathrm{K}$:

$$^{4}\mathrm{He} + \gamma \rightleftharpoons 2\,\mathrm{n} + 2\,\mathrm{p}$$

- This takes $\sim 28\,\mathrm{MeV}$ *per* $^{4}\mathrm{He}$
- $\Rightarrow$ again, gas energy is used to unbind nucleus
- takes (about) as much energy as was released before to burn $4\,^{1}\mathrm{H}$ to $^{4}\mathrm{He}$
  (not counting neutrino losses during hydrogen burning)
- $\Rightarrow \gamma_{\mathrm{ad}}$ drops
- $\Rightarrow$ possible instability of star (collapse)

Helium and Iron Dissociation



Bond, Arnett, Carr (1984)

Energy Transport Equation - Radiation Transport

- Define radiation flux H (per unit time, per unit area)

$$\mathrm{d}H = \mathrm{d}F / 4\pi r^2$$

- amount of radiation absorbed in a slab of thickness dr is

$$\mathrm{d}H = -\kappa H \rho \mathrm{d}r$$

  where κ is the *opacity* of the gas.

- for constant opacity and density, and assuming no heating or compression, we obtain the solution

$$H = H_0 e^{-\kappa \rho r}$$

Optical Depth and Stellar Radius

- The characteristic absorption length is therefore $\frac{1}{\kappa\rho}$ is also approximately the mean free path of the photon
- we define the quantity *optical depth* by

$$\mathrm{d}\tau = -\kappa\rho\,\mathrm{d}r$$

- define "photosphere" of star as where optical depth becomes of order unity (2/3):

$$\int_0^\infty \kappa\rho\,\mathrm{d}r$$

- on can show that in good approximation the radiation of a star can be described as surface with effective temperature T_{eff} given at the "effective" radius of the star, R_{eff} at $\tau = 2/3$

Opacity Sources

The main contributions to opacity are

- electron scattering (Thompson scattering)
  (relativistic: Compton scattering)
- free-free absorption (interaction of free electron with atom)
  (reverse: bremsstrahlung)
- bound-free absorption
  (reverse: recombination)
- bound-bound absorption
  (reverse: de-excitation)

Opacity Law

- opacity is function of T, ρ, composition
- we may parameterize

$$\kappa = \kappa_0(X)\,\rho^a T^b$$

- electron scattering (Thompson scattering)

$$\kappa_{\mathrm{es}} = \frac{\kappa_{\mathrm{es},0}}{\mu_{\mathrm{e}}} \approx \frac{1}{2}(1+X)\kappa_{\mathrm{es},0}$$

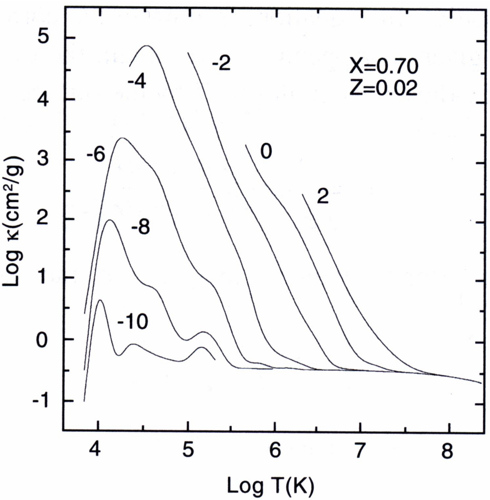
$\kappa_{\mathrm{es},0} = 0.4\,\mathrm{cm}^2\,\mathrm{g}^{-1},\ a = b = 0$

- free-free scattering (Kramers opacity law)

$$\kappa_{\mathrm{ff}} = \frac{\kappa_{\mathrm{ff},0}}{\mu_{\mathrm{e}}}\left\langle\frac{Z^2}{A}\right\rangle \rho T^{-7/2} \approx \frac{1}{2}\kappa_{\mathrm{ff},0}(1+X)\left\langle\frac{Z^2}{A}\right\rangle \rho T^{-7/2}$$

$\kappa_{\mathrm{ff},0} = 7.5\times10^{22}\,\mathrm{cm}^2\,\mathrm{g}^{-1},\ a = 1,\ b = -7/2$

Opacity



opacity for solar composition

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Equation of State and Opacity

- electron positron pair production for $T \gtrsim 10^9\,\mathrm{K}$
- iron dissociation for $T \gtrsim 7\times10^9\,\mathrm{K}$
- helium dissociation for $T \gtrsim 10^{10}\,\mathrm{K}$
- *optical depth*

$$\mathrm{d}\tau = -\kappa\rho\,\mathrm{d}r$$

- electron scattering (Thompson scattering)

$$\kappa_{\mathrm{es}} = \frac{\kappa_{\mathrm{es},0}}{\mu_{\mathrm{e}}} \approx \frac{1}{2}\kappa_{\mathrm{es},0}(1+X)$$

$$\kappa_{\mathrm{es},0} = 0.4\mathrm{cm}^2\,\mathrm{g}^{-1}$$

Stellar Evolution Project

- Bill Paxton's **EZ Stellar Evolution** code
  `http://www.kitp.ucsb.edu/~paxton/EZ-intro.html`
- Uses Linux `gfortran`
- g95 FORTRAN compiler can be downloaded for most platforms.
  `http://www.g95.org`