Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Announcement

William I. Fine Theoretical Physics Institute
University of Minnesota

The *3rd* Annual Irving and Edythe
MISEL *FAMILY* LECTURE

Professor Jim Peebles
Princeton University
Albert Einstein Professor of Science Emeritus

Finding the Big Bang

Public Lecture
Tuesday, September 23, 2008
7:00pm
Van Vleck Auditorium
Room 150, Tate Lab of Physics

Physics and Astronomy Colloquium
The Cosmological Tests
Wednesday, September 24, 2008
3:35pm
Room 131 Tate Lab of Physics

For more information visit www.ftpi.umn.edu/misel/

Announcement

Burning Neutron Stars

Alexander Heger

Friday, September 26, 15:00

The probably by far most common thermonuclear explosion to occur in nature is the explosion of a thin layer of material, about the height of the physics building, that has accumulated on the surface of a neutron star, about the size of Minneapolis, in a binary star system - Type I X-ray bursts. I show theoretical models for such outbursts, their very specific mode of nuclear burning unheard of in any other stellar system, as well as their much bigger cousins, the superbursts. I will discuss our current difficulty in understanding how those are made, and possible solutions.

Agenda

1. Recap
   - Summary
2. Energy Transport Equation
   - Radiative Energy Transport
   - Conductive Energy Transport
   - Rosseland Mean Opacity
3. Summary
   - Summary

Overview

Equation of State and Opacity

- electron positron pair production for $T \gtrsim 10^9\,\mathrm{K}$
- iron dissociation for $T \gtrsim 7\times10^9\,\mathrm{K}$
- helium dissociation for $T \gtrsim 10^{10}\,\mathrm{K}$
- *optical depth*

$$\mathrm{d}\tau = -\kappa\rho\,\mathrm{d}r$$

- electron scattering (Thompson scattering)

$$\kappa_{\mathrm{es}} = \frac{\kappa_{\mathrm{es},0}}{\mu_{\mathrm{e}}} \approx \frac{1}{2}\kappa_{\mathrm{es},0}(1+X)$$

$\kappa_{\mathrm{es},0} = 0.4\mathrm{cm}^2\,\mathrm{g}^{-1}$

Overview

Stellar Structure Equations - Temperature Gradient

stationary terms   time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \tag{2}$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_{\nu} - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{3}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \tag{4}$$

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) \tag{5}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Basic Estimates

- mean free path $l_{\mathrm{ph}} = \frac{1}{\kappa\rho}$
- for sun
  $\bar{\rho_\odot} = \frac{3\,\mathrm{M}_\odot}{4\pi {\mathrm{R}_\odot}^3} = 1.4\,\mathrm{g\,cm}^{-3}$,
  $\kappa \approx 0.4\,\mathrm{cm}^2\,\mathrm{g}^{-1}$
  $\Rightarrow l_{\mathrm{ph}} = 2\,\mathrm{cm}$
- using
  $T_{\mathrm{c}} \approx 10^7\,\mathrm{K}$,
  $T_{\mathrm{surf}} \approx 10^4\,\mathrm{K}$
  we may estimate

$$\frac{\Delta T}{\Delta r} \approx \frac{T_{\mathrm{c}} - T_{\mathrm{surf}}}{\mathrm{R}_\odot} \approx 1.4\times10^{-4}\,\mathrm{K\,cm}^{-1}$$

Basic Estimates

- note that $l_{\mathrm{ph}}/\mathrm{R}_\odot \approx 3\times10^{-11}$
- on a mean free path scale we have a temperature variation of $\Delta T \approx l_{\mathrm{ph}}\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right) \approx 3\times10^{-4}\,\mathrm{K}$
- with $u \sim T^4$ the anisotropy of radiation at $T = 10^7\,\mathrm{K}$ is

$$4\frac{\Delta T}{T} \sim 10^{-10}$$

- $\Rightarrow$ very close to thermal equilibrium
- at $T = 10^7\,\mathrm{K}$ this anisotropy is still 10^3 times bigger than flux at surface of $6\times10^{10}\,\mathrm{erg\,s^{-1}\,cm^{-2}}$
- $\Rightarrow$ even small anisotropy can carry large flux!

Diffusion of Radiative Energy

- diffusive flux given by

$$\mathbf{j} = -D\,\nabla n$$

- Diffusion coefficient

$$D = \frac{1}{3} v\, l_{\mathrm{ph}}$$

  determined by mean free path l_{ph} and velocity v of the particles

- for radiative flux H we replace:
  - n by energy density U (per unit volume), $U = aT^4$, and
  - v by the speed of light, c

Diffusion of Radiative Energy

- we are only interested in the radial component we may replace
  - $H_\mathrm{r} = |\mathbf{H}| = H$,
  - $\nabla U \rightarrow \frac{\partial U}{\partial r}$
- we the obtain

$$\frac{\partial U}{\partial r} = 4aT^3 \frac{\partial T}{\partial r}$$

- and for the flux

$$H = -\frac{4ac}{3} \frac{T^3}{\kappa\rho} \frac{\partial T}{\partial r}$$

- or in mass coordinate

$$H = -\frac{ac}{3\pi} \frac{T^3}{\kappa\rho^2 r^2} \frac{\partial T}{\partial m}$$

Heat Conduction

- formally we can write this as a heat conduction equation

$$\mathbf{H} = -k_{\mathrm{rad}} \nabla T$$

where we define

$$k_{\mathrm{rad}} = \frac{4ac}{3} \frac{T^3}{\kappa\rho}$$

- using $F = 4\pi r^2 H$ we can solve for

$$\frac{\partial T}{\partial r} = -\frac{3}{16\pi ac} \frac{\kappa\rho F}{r^2 T^4}$$

or, in mass coordinate

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi ac} \frac{\kappa F}{r^4 T^4}$$

Radiative Energy Transport

- if we divide by the equation for hydrostatic equilibrium

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

we obtain

$$\frac{\partial T}{\partial m} = \frac{3}{16\pi acG}\frac{\kappa F}{mT^3}\frac{\partial P}{\partial m}$$

- we may rewrite the temperature gradient due to radiation as

$$\nabla_{\mathrm{rad}} = \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa FP}{mT^4}$$

and obtain

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla_{\mathrm{rad}}$$

Quiz

- using the momentum equation including acceleration

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

find the appropriate expression for

$$\frac{\partial T}{\partial m}$$

Radiative Energy Transport

- if we divide by the momentum equation

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} = -\frac{Gm}{4\pi r^4}\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right]$$

we obtain

$$\frac{\partial T}{\partial m} = \frac{3}{16\pi acG}\frac{\kappa F}{mT^3}\frac{\partial P}{\partial m}\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right]^{-1}$$

- defining

$$\nabla_{\mathrm{rad}} = \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa FP}{mT^4}\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right]^{-1}$$

we write

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla_{\mathrm{rad}}\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right]$$

Conductive Energy Transport

- so far we only dealt with the radiative flux

$$\mathbf{H} = \mathbf{H}_{\mathrm{rad}}$$

- we can write the conductive flux in the same form

$$\mathbf{H}_{\mathrm{cd}} = -k_{\mathrm{cd}} \nabla T$$

- and add both up

$$\mathbf{H} = \mathbf{H}_{\mathrm{rad}} + \mathbf{H}_{\mathrm{cd}} = -(k_{\mathrm{rad}} + k_{\mathrm{cd}}) \nabla T$$

Conductive Energy Transport

- we can also formally write

$$k_{\mathrm{cd}} = \frac{4ac}{3}\frac{T^3}{\kappa_{\mathrm{cd}}\rho}$$

- and then have for the *total* flux

$$\mathbf{H} = -\frac{4\pi ac}{3}\frac{T^3}{\rho}\left(\frac{1}{\kappa_{\mathrm{rad}}} + \frac{1}{\kappa_{\mathrm{cd}}}\right)\nabla T$$

- and can define

$$\frac{1}{\kappa} = \frac{1}{\kappa_{\mathrm{rad}}} + \frac{1}{\kappa_{\mathrm{cd}}}$$

  to recover the original form

$$\mathbf{H} = -\frac{4\pi ac}{3}\frac{T^3}{\kappa\rho}\nabla T$$

Rosseland Mean Opacity

- for radiative transport, all quantities have to be considered a function of frequency, $l_{\nu,\mathrm{ph}}$, H_ν, D_ν, U_ν, κ_ν, etc.
- we then write the flux as

$$H_\nu = -D_\nu \nabla U_\nu$$

and

$$D_\nu = \frac{1}{3} c \, l_{\nu,\mathrm{ph}} = \frac{c}{3\kappa_\nu \rho}$$

- and the energy density as

$$U_\nu = \frac{4\pi}{c} B(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/k_\mathrm{B} T} - 1}$$

where $B(\nu, T)$ is the Planck function for intensity

Rosseland Mean Opacity

- we may hence write

$$\nabla U_\nu = \frac{4\pi}{c}\frac{\partial B}{\partial T}\nabla T$$

- we obtain the total flux by integration over all frequencies

$$H = -\left[\frac{4\pi}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B}{\partial T}\,\mathrm{d}\nu\right]\nabla T$$

- this is a conduction equation

$$\mathbf{H} = -k_{\mathrm{rad}}\nabla T$$

with

$$k_{\mathrm{rad}} = \frac{4\pi}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B}{\partial T}\,\mathrm{d}\nu$$

Rosseland Mean Opacity

- and from

$$k_{\mathrm{rad}} = \frac{4ac}{3}\frac{T^3}{\kappa\rho}$$

  we can define the Rosseland mean opacity

$$\frac{1}{\kappa} = \frac{\pi}{acT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B}{\partial T}\,\mathrm{d}\nu$$

- since

$$\frac{acT^3}{\pi} = \int_0^\infty \frac{\partial B}{\partial T}\,\mathrm{d}\nu$$

  the Rosseland mean opacity is the harmonic mean.
- considering

$$H_\nu = -\left(\frac{1}{\kappa_\nu}\frac{\partial B}{\partial T}\right)\frac{4\pi}{3\rho}\nabla T$$

  we see that frequencies of maximum energy flux are favored in the mean.

Overview

Summary

- energy flux and effective opacity

$$\mathbf{H} = -\frac{4\pi ac}{3}\frac{T^3}{\kappa\rho}\nabla T\,, \quad \frac{1}{\kappa} = \frac{1}{\kappa_{\mathrm{rad}}} + \frac{1}{\kappa_{\mathrm{cd}}}$$

- temperature gradient in hydrostatic star

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla$$

- where for radiation we have

$$\nabla = \nabla_{\mathrm{rad}} = \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa FP}{mT^4}$$

- Rosseland mean opacity

$$\frac{1}{\kappa} = \frac{\pi}{acT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B}{\partial T}\,\mathrm{d}\nu$$