Astrophysics I: Stars and Stellar Evolution

AST 4001

Alexander Heger[1,2,3]

[1]School of Physics and Astronomy
University of Minnesota

[2]Theoretical Astrophysics Group, T-6
Los Alamos National Laboratory

[3]Department of Astronomy and Astrophysics
University of California at Santa Cruz

Stars and Stellar Evolution, Fall 2008

Announcement

William I. Fine
Theoretical Physics Institute
University of Minnesota

The *3rd* Annual Irving and Edythe
MISEL *FAMILY* LECTURE

Professor Jim Peebles
Princeton University
Albert Einstein Professor of Science Emeritus
Finding the Big Bang

Public Lecture
Tuesday, September 23, 2008
7:00pm
Van Vleck Auditorium
Room 150, Tate Lab of Physics

Physics and Astronomy Colloquium
The Cosmological Tests
Wednesday, September 24, 2008
3:35pm
Room 131 Tate Lab of Physics

For more information visit: www.ftpi.umn.edu/misel/

Announcement

Burning Neutron Stars

Alexander Heger

Friday, September 26, 15:00

The probably by far most common thermonuclear explosion to occur in nature is the explosion of a thin layer of material, about the height of the physics building, that has accumulated on the surface of a neutron star, about the size of Minneapolis, in a binary star system - Type I X-ray bursts. I show theoretical models for such outbursts, their very specific mode of nuclear burning unheard of in any other stellar system, as well as their much bigger cousins, the superbursts. I will discuss our current difficulty in understanding how those are made, and possible solutions.

Agenda

1. Recap
   - Summary
   - Optical depth and Mean Free Path

2. Stellar Structure Equations
   - Energy Equation

Overview

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Summary

- energy flux and effective opacity

$$\mathbf{H} = -\frac{4\pi ac}{3}\frac{T^3}{\kappa\rho}\nabla T\,, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_{\mathrm{rad}}} + \frac{1}{\kappa_{\mathrm{cd}}}$$

- temperature gradient in hydrostatic star

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla$$

- where for radiation we have

$$\nabla = \nabla_{\mathrm{rad}} = \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa FP}{mT^4}$$

- Rosseland mean opacity

$$\frac{1}{\kappa} = \frac{\pi}{acT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B}{\partial T}\,\mathrm{d}\nu$$

Notes on Photon Mean Free Path an Optical Depth

- recall

$$l_{\mathrm{ph}} = \frac{1}{\kappa\rho} \quad \text{and} \quad \tau(r) = -\int_r^\infty \kappa\rho\,\mathrm{d}r$$

- $\Rightarrow \tau$ is approximation for number of mean free paths
- diffusion time is proportional

$$\tau_{\mathrm{diff}} \sim \frac{\mathrm{length}^2}{D}$$

- Questions:
  1. What is the optical depth at the center of the sun? (assume $l_{\mathrm{ph}} = 2\,\mathrm{cm}$)
  2. How long does a photon take to get from the center to the surface of the sun?

Overview

Stellar Structure Equations - Energy Equation

stationary terms time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad (1)$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \quad (2)$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \quad (3)$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \quad (4)$$

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) \quad (5)$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Stellar Structure Equations - Energy Equation

- Recall the energy equation in the form

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_{\nu} + \dot{u} + P\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \varepsilon_{\mathrm{nuc}} - \varepsilon_{\nu} + \dot{u} - \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}$$

- recall first law of thermodynamics

$$\mathrm{d}q = \mathrm{d}u + P\mathrm{d}\left(\frac{1}{\rho}\right) = \mathrm{d}u - \frac{P}{\rho^2}\mathrm{d}\rho$$

- assume general equation of state

$$\rho = \rho(P, T)\,, \quad u = u(\rho, T)$$

Stellar Structure Equations - Energy Equation - EOS

- Defining

$$\alpha = \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_T, \quad \delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P$$

- we can then write the equation of state in the form

$$\frac{\mathrm{d}\rho}{\rho} = \left(\frac{\partial \rho}{\partial P}\right)_T \frac{\mathrm{d}P}{\rho} + \left(\frac{\partial \rho}{\partial T}\right)_P \frac{\mathrm{d}T}{\rho} = \alpha \frac{\mathrm{d}P}{P} - \delta \frac{\mathrm{d}T}{T}$$

- the specific heats are defined as

$$c_P = \left(\frac{\mathrm{d}q}{\mathrm{d}T}\right)_P = \left(\frac{\partial u}{\partial T}\right)_P - \frac{P}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_P$$

$$c_V = \left(\frac{\mathrm{d}q}{\mathrm{d}T}\right)_V = \left(\frac{\partial u}{\partial T}\right)_V$$

(Note: keeping volume V constant is same as keeping density ρ constant)

Stellar Structure Equations - Energy Equation

- One can show that

$$c_P = c_V + \frac{P\delta^2}{\rho T \alpha}$$

- after some algebra and calculus

$$\mathrm{d}q = \mathrm{d}u + P\mathrm{d}\left(\frac{1}{\rho}\right) = \mathrm{d}u - \frac{P}{\rho^2}\mathrm{d}\rho$$

becomes

$$\mathrm{d}q = c_P\,\mathrm{d}T + \frac{\delta}{\rho}\,\mathrm{d}P$$

- we then obtain the Energy equation in the form

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_{\nu} - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t}$$

- *Reference: Kippenhahn & Weigert, 1990, pp. 19-21*