Astrophysics I: Stars and Stellar Evolution AST 4001

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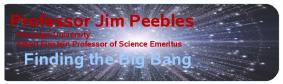
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Stars and Stellar Evolution, Fall 2008

Announcement



The *3rd* Annual Irving and Edythe MISEL *FAMILY* LECTURE



Public Lecture Tuesday, September 23, 2008 7:00pm

Van Vleck Auditorium Room 150, Tate Lab of Physics Physics and Astronomy Colloquium The Cosmological Tests Wednesday, September 24, 2008 3:35pm Room 131 Tate Lab of Physics

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For more information and implementation advimisel/

Announcement

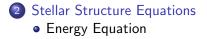
Burning Neutron Stars Alexander Heger Friday, September 26, 15:00

The probably by far most common thermonuclear explosion to occur in nature is the explosion of a thin layer of material, about the height of the physics building, that has accumulated on the surface of a neutron star, about the size of Minneapolis, in a binary star system - Type I X-ray bursts. I show theoretical models for such outbursts, their very specific mode of nuclear burning unheard of in any other stellar system, as well as their much bigger cousins, the superbursts. I will discuss our current difficulty in understanding how those are made, and possible solutions.





- Summary
- Optical depth and Mean Free Path







- Summary
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Summary

• energy flux and effective opacity

$$\mathbf{H} = -\frac{4\pi ac}{3} \frac{T^3}{\kappa \rho} \, \nabla T \,, \quad \frac{1}{\kappa} = \frac{1}{\kappa_{\mathsf{rad}}} + \frac{1}{\kappa_{\mathsf{cd}}}$$

• temperature gradient in hydrostatic star

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla$$

• where for radiation we have

$$\nabla = \nabla_{\mathsf{rad}} = \left(\frac{\mathsf{d}\,\,\mathsf{ln}\,\,T}{\mathsf{d}\,\,\mathsf{ln}\,P}\right)_{\mathsf{rad}} = \frac{3}{16\pi\,\mathsf{acG}}\frac{\kappa FP}{mT^4}$$

• Rosseland mean opacity

$$\frac{1}{\kappa} = \frac{\pi}{acT^3} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B}{\partial T} \,\mathrm{d}\nu$$

Summary Optical depth and Mean Free Path

Notes on Photon Mean Free Path an Optical Depth

recall

$$I_{\mathsf{ph}} = rac{1}{\kappa
ho}$$
 and $au(r) = -\int_r^\infty \kappa
ho\,\mathsf{d}r$

 $\bullet \ \Rightarrow \tau$ is approximation for number of mean free paths

• diffusion time is proportional

$$au_{
m diff} \sim rac{
m length^2}{D}$$

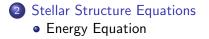
• Questions:

- What is the optical depth at the center of the sun? (assume l_{ph} = 2 cm)
- Output How long does a photon take to get from the center to the surface of the sun?

Overview

1 Recap

- Summary
- Optical depth and Mean Free Path



Energy Equation

Stellar Structure Equations - Energy Equation

stationary terms

time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$
(1)
$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2} \frac{\partial^2 r}{\partial t^2}$$
(2)
$$\frac{\partial F}{\partial m} = \varepsilon_{\text{nuc}} - \varepsilon_{\nu} - c_{P} \frac{\partial T}{\partial t} + \frac{\delta}{\rho} \frac{\partial P}{\partial t}$$
(3)
$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P} \nabla \left[1 + \frac{r^2}{Gm} \frac{\partial^2 r}{\partial t^2} \right]$$
(4)

$$\frac{\partial N_i}{\partial t} = f_i(\rho, T, \mathbf{X}) \tag{5}$$

where $\mathbf{X} = \{X_1, X_2, \dots, X_i, \dots\}$.

Stellar Structure Equations - Energy Equation

Stellar Structure Equations

Recap

• Recall the energy equation in the form

$$\frac{\partial F}{\partial m} = \varepsilon_{\rm nuc} - \varepsilon_{\nu} + \dot{u} + P \frac{\partial}{\partial t} \left(\frac{1}{\rho}\right) = \varepsilon_{\rm nuc} - \varepsilon_{\nu} + \dot{u} - \frac{P}{\rho^2} \frac{\partial \rho}{\partial t}$$

Energy Equation

• recall first law of thermodynamics

$$dq = du + Pd\left(\frac{1}{\rho}\right) = du - \frac{P}{\rho^2}d
ho$$

assume general equation of state

$$\rho = \rho(P, T), \quad u = u(\rho, T)$$

Energy Equation

Stellar Structure Equations - Energy Equation - EOS

Defining

$$\alpha = \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_{T}, \quad \delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P}$$

• we can then write the equation of state in the form

$$\frac{\mathrm{d}\rho}{\rho} = \left(\frac{\partial\rho}{\partial P}\right)_{T} \frac{\mathrm{d}P}{\rho} + \left(\frac{\partial\rho}{\partial T}\right)_{P} \frac{\mathrm{d}T}{\rho} = \alpha \frac{\mathrm{d}P}{P} - \delta \frac{\mathrm{d}T}{T}$$

• the specific heats are defined as

$$c_{P} = \left(\frac{\mathrm{d}q}{\mathrm{d}T}\right)_{P} = \left(\frac{\partial u}{\partial T}\right)_{P} - \frac{P}{\rho^{2}}\left(\frac{\partial \rho}{\partial T}\right)_{P}$$
$$c_{V} = \left(\frac{\mathrm{d}q}{\mathrm{d}T}\right)_{V} = \left(\frac{\partial u}{\partial T}\right)_{V}$$

(Note: keeping volume V constant is same as keeping density ρ constant)

Energy Equation

Stellar Structure Equations - Energy Equation

• One can show that

$$c_P = c_V + \frac{P\delta^2}{\rho T\alpha}$$

• after some algebra and calculus

$$dq = du + Pd\left(\frac{1}{\rho}\right) = du - \frac{P}{\rho^2}d\rho$$

becomes

$$\mathrm{d}q = c_P \,\mathrm{d}T + \frac{\delta}{\rho} \,\mathrm{d}P$$

• we then obtain the Energy equation in the form

$$\frac{\partial F}{\partial m} = \varepsilon_{\rm nuc} - \varepsilon_{\nu} - c_{P} \frac{\partial T}{\partial t} + \frac{\delta}{\rho} \frac{\partial P}{\partial t}$$

Reference: Kippenhahn & Weigert, 1990, pp. 19-21