Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Announcement

Burning Neutron Stars

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The probably by far most common thermonuclear explosion to occur in nature is the explosion of a thin layer of material, about the height of the physics building, that has accumulated on the surface of a neutron star, about the size of Minneapolis, in a binary star system - Type I X-ray bursts. I show theoretical models for such outbursts, their very specific mode of nuclear burning unheard of in any other stellar system, as well as their much bigger cousins, the superbursts. I will discuss our current difficulty in understanding how those are made, and possible solutions.

Agenda

1. Recap
   - Stellar Structure Equations
   - Thermal Adjustment Time of a Star

2. Convection
   - What is Convection?
   - Stability Against Convection

Overview

1. Recap
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Stellar Structure Equations - Energy Equation

stationary terms   time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \qquad (1)$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \qquad (2)$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P\frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \qquad (3)$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla\left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \qquad (4)$$

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) \qquad (5)$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Stellar Structure Equations - Energy Equation

- Recall the energy equation in the form

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_{\nu} + \dot{u} + P\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \varepsilon_{\mathrm{nuc}} - \varepsilon_{\nu} + \dot{u} - \frac{P}{\rho^2}\frac{\partial\rho}{\partial t}$$

- recall first law of thermodynamics

$$\mathrm{d}q = \mathrm{d}u + P\mathrm{d}\left(\frac{1}{\rho}\right) = \mathrm{d}u - \frac{P}{\rho^2}\mathrm{d}\rho$$

- assume general equation of state

$$\rho = \rho(P,T)\,, \quad u = u(\rho,T)$$

Stellar Structure Equations - Energy Equation - EOS

- Defining

$$\alpha = \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_T , \quad \delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P$$

- we can then write the equation of state in the form

$$\frac{d\rho}{\rho} = \left(\frac{\partial \rho}{\partial P}\right)_T \frac{dP}{\rho} + \left(\frac{\partial \rho}{\partial T}\right)_P \frac{dT}{\rho} = \alpha \frac{dP}{P} - \delta \frac{dT}{T}$$

- the specific heats are defined as

$$c_P = \left(\frac{dq}{dT}\right)_P = \left(\frac{\partial u}{\partial T}\right)_P - \frac{P}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_P$$

$$c_V = \left(\frac{dq}{dT}\right)_V = \left(\frac{\partial u}{\partial T}\right)_V$$

(Note: keeping volume V constant is same as keeping density ρ constant)

Stellar Structure Equations - Energy Equation

- One can show that

$$c_P = c_V + \frac{P\delta^2}{\rho T \alpha}$$

- after some algebra and calculus

$$\mathrm{d}q = \mathrm{d}u + P\mathrm{d}\left(\frac{1}{\rho}\right) = \mathrm{d}u - \frac{P}{\rho^2}\mathrm{d}\rho$$

becomes

$$\mathrm{d}q = c_P\,\mathrm{d}T + \frac{\delta}{\rho}\,\mathrm{d}P$$

- we then obtain the Energy equation in the form

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t}$$

- *Reference: Kippenhahn & Weigert, 1990, pp. 19-21*

Thermal Adjustment Time of a Star

- We can write the radiative energy transport in the form

$$F = -\sigma^* \frac{\partial T}{\partial m}$$

where we define

$$\sigma^* = \frac{64\pi^2 ac T^3 r^4}{3\kappa}$$

- let us use $u = c_V T$ in

$$\frac{\partial F}{\partial m} = \varepsilon - \frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}$$

to obtain

$$\frac{\partial}{\partial m}\left(\sigma^* \frac{\partial T}{\partial m}\right) - c_V \frac{\partial T}{\partial t} = -\left[\varepsilon + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}\right]$$

Thermal Adjustment Time of a Star

- neglecting the term

$$\varepsilon + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}$$

  (point source in the center, hydrostatic star)
  we have an equation of the form

$$\frac{\partial}{\partial x}\left(\sigma \frac{\partial T}{\partial x}\right) = c\frac{\partial T}{\partial t}$$

  which is the equation of heat conduction along a rod.

- We may estimate a characteristic "thermal adjustment time scale" by

$$\tau_{\mathrm{adj}} = \frac{c}{\sigma} d^2$$

  where d is a characteristic length.

Thermal Adjustment Time of a Star

- for our star a crude estimate is (in mass coordinate) $d \approx M$ and we find

$$\tau_{\mathrm{adj}} = \frac{c_V M^2}{\bar{\sigma^*}}$$

  with an average $\bar{\sigma^*}$ over the entire star

- on the other hand, from

$$F = -\sigma^* \frac{\partial T}{\partial m}$$

  we may estimate using an average temperature $\bar{T}$:

$$L \approx \bar{\sigma^*}\,\bar{T}/M$$

  and obtain

$$\tau_{\mathrm{adj}} \approx \frac{c_V \bar{T} M}{L} \approx \frac{U}{L} = \tau_{\mathrm{KH}}$$

  where U is the internal energy of the star.

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Solar Convection

(solar convection)

Solar Convection (3D simulation)

(solar convection)

Convection (3D simulation)

(solar convection)

Convection (3D simulation)

(3D convection movie)

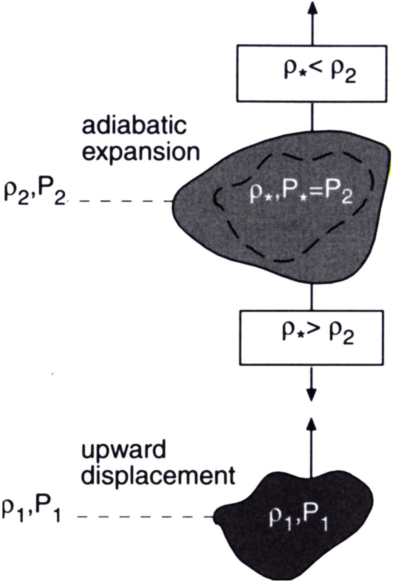
Convection (3D simulation)

(3D convection movie)

Convection in Stars

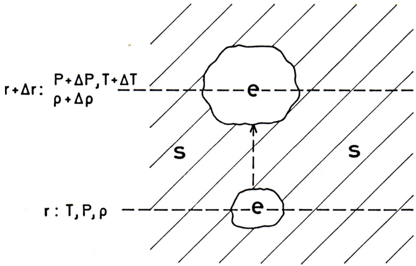
- For heated water
  - layer (bubble) heated at the bottom
  - $\Rightarrow$ expands
  - $\Rightarrow$ lighter than layer above
  - $\Rightarrow$ Rayleigh-Taylor instability (bubble rises)
- In stars we have to consider
  - stratified layer due to compression of gas
    $\Rightarrow$ bottom layers more dense
    can they rise when heated?
  - composition gradient due to nuclear burning
    $\Rightarrow$ bottom layers more dense

Rising Bubble Model



- assume pressure equilibrium
- assume no heat exchange (adiabatic expansion)
- if perturbed density is lower (higher) than the surrounding, the bubble will continue rising (drop back).

Rising Bubble Model



In the following we distinguish between

- the (rising) element "e"
- the surroundings "s"

We compare locations r and $r + \Delta r$.

$$\Delta\rho = \left[\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{\mathrm{e}} - \left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{\mathrm{s}}\right] \Delta r$$

Stability requires hence

$$\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{\mathrm{e}} - \left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{\mathrm{s}} > 0$$

Equations for local stability

Equation of state

$$\frac{\mathrm{d}\rho}{\rho} = \alpha \frac{\mathrm{d}P}{P} - \delta \frac{\mathrm{d}T}{T} + \varphi \frac{\mathrm{d}\mu}{\mu}$$

Where

$$\alpha = \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_{T,\mu}, \quad \delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}, \quad \varphi = \left(\frac{\partial \ln \rho}{\partial \ln \mu}\right)_{P,T},$$

Assuming there is no composition change in the element we obtain

$$\left(\frac{\alpha}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right)_\mathrm{e} - \left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_\mathrm{e} - \left(\frac{\alpha}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right)_\mathrm{s} + \left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_\mathrm{s} - \left(\frac{\varphi}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}r}\right)_\mathrm{s} > 0$$

Equations for local stability

- for pressure equilibrium the two pressure terms cancel out,

$$\left(\frac{\alpha}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right)_{\mathrm{e}} = \left(\frac{\alpha}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right)_{\mathrm{s}}$$

  and we are left with

$$-\left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{e}} + \left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{s}} - \left(\frac{\varphi}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}r}\right)_{\mathrm{s}} > 0$$

- Multiplying this equation by the pressure scale height

$$H_P := -\frac{\mathrm{d}r}{\mathrm{d}\ln P} = -P\frac{\mathrm{d}r}{\mathrm{d}P} > 0$$

  we obtain

$$\left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_{\mathrm{s}} < \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_{\mathrm{e}} + \frac{\varphi}{\delta}\left(\frac{\mathrm{d}\ln\mu}{\mathrm{d}\ln P}\right)_{\mathrm{s}}$$

- Note that for hydrostatic equilibrium $H_P = \frac{P}{g\rho}$, $g = Gm/r^2$

Equations for local stability

Introducing

$$\nabla = \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_{\mathrm{s}}, \quad \nabla_{\mathrm{e}} = \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_{\mathrm{e}}, \quad \nabla_{\mu} = \left(\frac{\mathrm{d}\ \ln \mu}{\mathrm{d}\ \ln P}\right)_{\mathrm{s}},$$

We get

$$\nabla < \nabla_{\mathrm{e}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

Let us assume the element moves adiabatic, and the surroundings is radiative, and we define

$$\nabla_{\mathrm{ad}} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_{\mathrm{ad}}, \quad \nabla_{\mathrm{rad}} = \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_{\mathrm{rad,star}}$$