Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

Stellar Structure Equations - Convection

stationary terms time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \tag{2}$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{3}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \tag{4}$$

$$\frac{\partial X_i}{\partial t} = f_i(\rho, T, \mathbf{X}) \tag{5}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Convection in O Shell Bunring

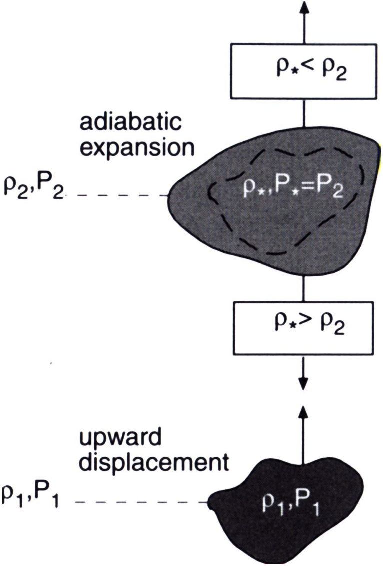
Courtesy of Casey Maekin (University of Arizona)

(4 movies, external, on web page)

Convection in Stars

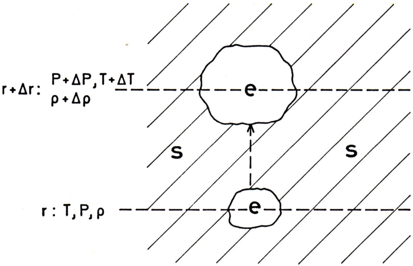
- For heated water
  - layer (bubble) heated at the bottom
  - $\Rightarrow$ expands
  - $\Rightarrow$ lighter than layer above
  - $\Rightarrow$ Rayleigh-Taylor instability (bubble rises)
- In stars we have to consider
  - stratified layer due to compression of gas
    $\Rightarrow$ bottom layers more dense
    can they rise when heated?
  - composition gradient due to nuclear burning
    $\Rightarrow$ bottom layers more dense

Rising Bubble Model



- assume pressure equilibrium
- assume no heat exchange (adiabatic expansion)
- if perturbed density is lower (higher) than the surrounding, the bubble will continue rising (drop back).

Rising Bubble Model



In the following we distinguish between

- the (rising) element "e"
- the surroundings "s"

We compare locations r and $r + \Delta r$.

$$\Delta\rho = \left[\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{\mathrm{e}} - \left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{\mathrm{s}}\right] \Delta r$$

Stability requires hence

$$\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{\mathrm{e}} - \left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{\mathrm{s}} > 0$$

Equation for local stability

Equation of state

$$\frac{\mathrm{d}\rho}{\rho} = \alpha \frac{\mathrm{d}P}{P} - \delta \frac{\mathrm{d}T}{T} + \varphi \frac{\mathrm{d}\mu}{\mu}$$

Where

$$\alpha = \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_{T,\mu}, \ \delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}, \ \varphi = \left(\frac{\partial \ln \rho}{\partial \ln \mu}\right)_{P,T},$$

Assuming there is no composition change in the element we obtain

$$\left(\frac{\alpha}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right)_{\mathrm{e}} - \left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{e}} - \left(\frac{\alpha}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right)_{\mathrm{s}} + \left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{s}} - \left(\frac{\varphi}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}r}\right)_{\mathrm{s}} > 0$$

Equation for local stability

- for pressure equilibrium the two pressure terms cancel out,

$$\left(\frac{\alpha}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right)_{\mathrm{e}} = \left(\frac{\alpha}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right)_{\mathrm{s}}$$

  and we are left with

$$-\left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{e}} + \left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{s}} - \left(\frac{\varphi}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}r}\right)_{\mathrm{s}} > 0$$

- Multiplying this equation by the pressure scale height

$$H_P := -\frac{\mathrm{d}r}{\mathrm{d}\ln P} = -P\frac{\mathrm{d}r}{\mathrm{d}P}$$

  where we note that for hydrostatic equilibrium

$$g = -\frac{Gm}{r^2}\,, \quad H_P = -\frac{P}{g\rho} > 0$$

Equation for local stability

We then obtain

$$\left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_\mathrm{s} < \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_\mathrm{e} + \frac{\varphi}{\delta}\left(\frac{\mathrm{d}\ \ln \mu}{\mathrm{d}\ \ln P}\right)_\mathrm{s}$$

Introducing

$$\nabla = \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_\mathrm{s}\ ,\ \nabla_\mathrm{e} = \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_\mathrm{e}\ ,\ \nabla_\mu = \left(\frac{\mathrm{d}\ \ln \mu}{\mathrm{d}\ \ln P}\right)_\mathrm{s}\ ,$$

we can write this in the simple form

$$\nabla < \nabla_\mathrm{e} + \frac{\varphi}{\delta}\nabla_\mu$$

Overview

1. Recap
   - Stellar Structure Equations
   - What is Convection?
   - Stability against convection

2. Convection
   - Convection Criteria
   - Temperature Gradients

Equations for local stability

If we assume the element moves adiabatically,

$$\nabla_{\mathrm{e}} = \nabla_{\mathrm{ad}} \equiv \left(\frac{\partial \ln T}{\partial \ln P} \right)_{\mathrm{ad}}$$

and the surroundings is radiative, that is

$$\nabla = \nabla_{\mathrm{rad}} \equiv \left(\frac{\mathrm{d} \ln T}{\mathrm{d} \ln P} \right)_{\mathrm{rad,star}}$$

we can now write

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta} \nabla_{\mu}$$

Ledoux and Schwarzschild criteria

- In general, for convection to set in, the temperature gradient has to be steep enough to overcome the composition gradient

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

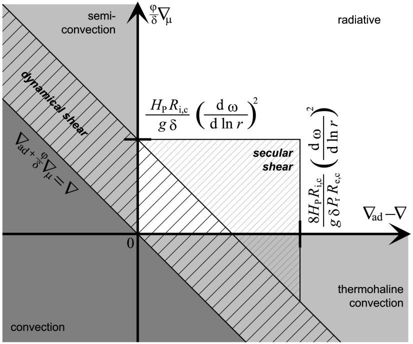
  This is the **Ledoux criterion** for dynamical stability.

- If no gradient in μ is present, or it is neglected (“convection mixed” - removed a posteriori) we obtain

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}}$$

  This is the **Schwarzschild criterion** for dynamical stability.

Local dynamical stability



Domains of dynamical (in)stability

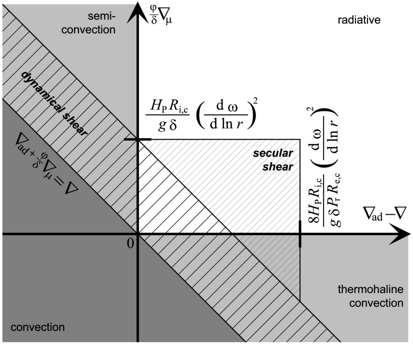
- **Ledoux Convection** where

$$\nabla_{rad} > \nabla_{ad} + \frac{\varphi}{\delta}\nabla_\mu$$

- **Schwarzschild Convection** where

$$\nabla_{rad} > \nabla_{ad}$$

Local dynamical stability



Domains of secular (in)stability

- **Semiconvection** where

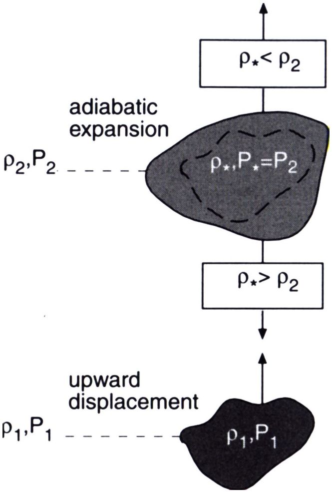
$$\frac{\varphi}{\delta}\nabla_\mu > 0, \nabla_{\rm rad} < \nabla_{\rm ad} + \frac{\varphi}{\delta}\nabla_\mu$$

- **Thermohaline Convection** where

$$\frac{\varphi}{\delta}\nabla_\mu < 0, \nabla_{\rm rad} < \nabla_{\rm ad} + \frac{\varphi}{\delta}\nabla_\mu$$

(salt finger instability)

Temperature Gradients



- What happens when an instability occurs?
- How does the temperature of the bubble and the surroundings evolve?
- At what rate is energy transported?

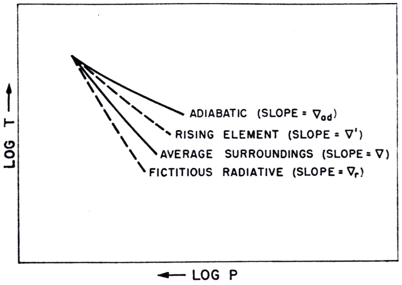
Temperature Gradients

- Without convection, - mass elements held in place - the surroundings will have the radiative temperature gradient.
- Without heat exchange, the bubbles will move **adiabatically**,

$$\frac{\partial S}{\partial t} = 0$$

  i.e., their entropy S does not change.
- In praxis, they will be *hotter* than their surroundings (assuming chemical homogeneity), and therefore *lose heat*.
- If energy transport in convection zone is efficient, the surroundings will also assume a temperature gradient very close to adiabatic.

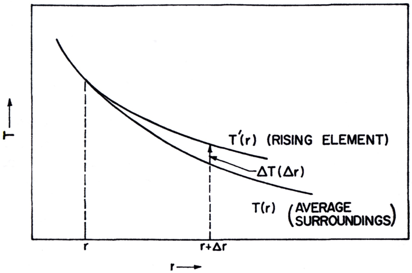
Temperature Gradients



The four gradients in an convective environment

- adiabatic slope
- rising bubble (loosing some heat)
- average slope of the surrounding
  steeper then bubble slope to allow driving of convection
- radiative gradient that were present without convection

Convective Temperature Gradients



Yet another view, showing temperature as a function of radius

Note: In convection zone we have a very small deviation of the temperature gradient form the adiabatic temperature gradient, $\nabla \lesssim \nabla_{\mathrm{ad}}$.