Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - Stellar Structure Equations
   - Convection Criteria
   - Temperature Gradients

2. Convection
   - Schematic Picture
   - Convective Overshooting and Penetration
   - Summary

Stellar Structure Equations - Convection

stationary terms time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \tag{2}$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{3}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \tag{4}$$

$$\frac{\partial X_i}{\partial t} = f_i(\rho, T, \mathbf{X}) \tag{5}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Equation for Local Stability

We have derived

$$\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_\mathrm{e} - \left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_\mathrm{s} > 0 \quad \Rightarrow \quad \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_\mathrm{s} < \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_\mathrm{e} + \frac{\varphi}{\delta}\left(\frac{\mathrm{d}\ln \mu}{\mathrm{d}\ln P}\right)_\mathrm{s}$$

Introducing

$$\nabla = \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_\mathrm{s}, \; \nabla_\mathrm{e} = \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_\mathrm{e}, \; \nabla_\mu = \left(\frac{\mathrm{d}\ln \mu}{\mathrm{d}\ln P}\right)_\mathrm{s},$$

we can write this in the simple form

$$\nabla < \nabla_\mathrm{e} + \frac{\varphi}{\delta}\nabla_\mu$$

Equations for local stability

If we assume the element moves adiabatically,

$$\nabla_{\mathrm{e}} = \nabla_{\mathrm{ad}} \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)_{\mathrm{ad}}$$

and the surroundings is radiative, that is

$$\nabla = \nabla_{\mathrm{rad}} \equiv \left(\frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln P}\right)_{\mathrm{rad,star}}$$

we can now write

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

Ledoux and Schwarzschild criteria

- In general, for convection to set in, the temperature gradient has to be steep enough to overcome the composition gradient

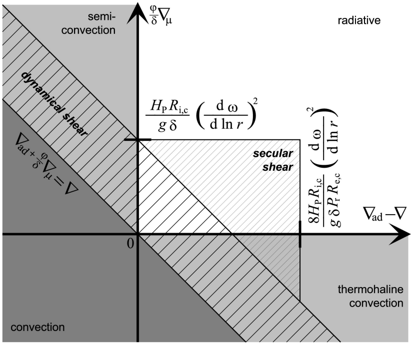
$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

  This is the **Ledoux criterion** for dynamical stability.
- If no gradient in μ is present, or it is neglected ("convection mixed" - removed a posteriori) we obtain

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}}$$

  This is the **Schwarzschild criterion** for dynamical stability.

Local dynamical stability



Domains of dynamical (in)stability

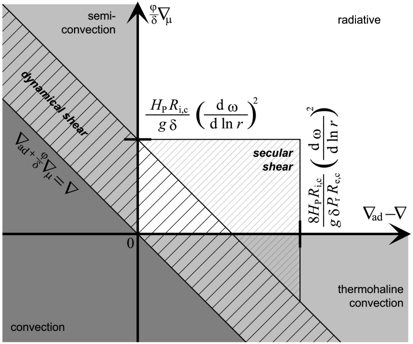
- **Ledoux Convection** where

$$\nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

- **Schwarzschild Convection** where

$$\nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}}$$

Local dynamical stability



Domains of secular (in)stability

- **Semiconvection** where

$$\frac{\varphi}{\delta}\nabla_\mu > 0, \nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_\mu$$

- **Thermohaline Convection** where

$$\frac{\varphi}{\delta}\nabla_\mu < 0, \nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_\mu$$

(salt finger instability)

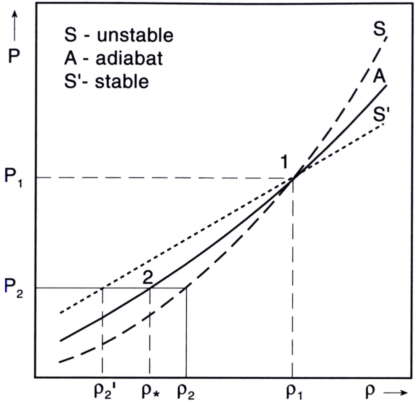
Temperature Gradients

- Without convection, - mass elements held in place - the surroundings will have the radiative temperature gradient.
- Without heat exchange, the bubbles will move **adiabatically**,

$$\frac{\partial S}{\partial t} = 0$$

  i.e., their entropy S does not change.
- In praxis, they will be *hotter* than their surroundings (assuming chemical homogeneity), and therefore *lose heat*.
- If energy transport in convection zone is efficient, the surroundings will also assume a temperature gradient very close to adiabatic.

Density Gradients



Generally, for a chemically homogeneous stratification,

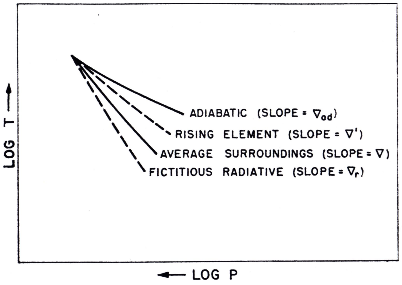
- a super-adiabatic gradient is unstable

$$\nabla > \nabla_{\mathrm{ad}}$$

- a sub-adiabatic gradient is stable

$$\nabla < \nabla_{\mathrm{ad}}$$

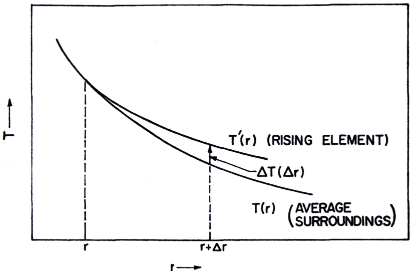
Temperature Gradients



The four gradients in an convective environment

- adiabatic slope
- rising bubble (loosing some heat)
- average slope of the surrounding
  steeper then bubble slope to allow driving of convection
- radiative gradient that were present without convection

Convective Temperature Gradients



Yet another view, showing temperature as a function of radius

Note: In convection zone we have a very small deviation of the temperature gradient form the adiabatic temperature gradient, $\nabla \lesssim \nabla_{\rm ad}$.

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Convection - **Very** Schematic Picture

- Consider a convection zone of mass M_c and internal energy u at radius r, thickness $r_c \lesssim r$ and a local luminosity of F.
- as energy is transferred crosses, a mass element is heated by ΔT
- its density changes by $\Delta\rho/\rho = -\delta\,\Delta T/T$ – it expands.
- assume an average velocity, v_c. It can be estimated as

$$v_c \sim \sqrt{\frac{GM}{r^2}\frac{\Delta\rho}{\rho}r_c}$$

- the time required for the element to cross the shell is hence r_c/v_c
- $\Rightarrow$ an element absorbs an energy of $F \times (r_c/v_c)$ before it moves away.

Convection - **Very** Schematic Picture

- The corresponding rise in T of the element can be estimated as

$$\frac{\Delta T}{T} \sim \frac{F \times (r_c/v_c)}{uM_c}$$

- We obtain for v_c:

$$v_c \sim \sqrt{\frac{Gm}{r^2}\frac{(-\Delta\rho)}{\rho}r_c} \sim \sqrt{\frac{Gm}{r^2}\delta\frac{\Delta T}{T}r_c}$$

- and

$$\left(\frac{\Delta T}{T}\right)^{3/2} \sim \frac{Fr}{uM_c}\sqrt{\frac{r_c}{Gm\delta}}$$

Convection - Rough Estimates

- replace r, r_c by R
- replace m, M_c by M
- replace uM by U
- replace F by L
- assume $\delta \sim 1$
- we obtain

$$\left(\frac{\Delta T}{T}\right)^{3/2} \sim \frac{L}{U}\sqrt{\frac{R^3}{GM}} \sim \frac{\tau_\mathrm{dynamic}}{\tau_\mathrm{thermal}}$$

$$\frac{\Delta T}{T} \sim \left(\frac{\tau_\mathrm{dynamic}}{\tau_\mathrm{thermal}}\right)^{2/3} \sim 10^{-8}$$

Convection - Notes

- At the surface, however, $uM \ll U$ and this approximation becomes invalid.
- That is, one can have a strongly *super adiabatic* stratification i.e., temperature gradient is much steeper than adiabatic
- In general the temperature gradient in convective regions is very close to adiabatic, just slightly steeper to drive convection.
- In convective regions we hence have

$$\frac{\mathrm{d}S}{\mathrm{d}r} \lesssim 0$$

and in radiative regions

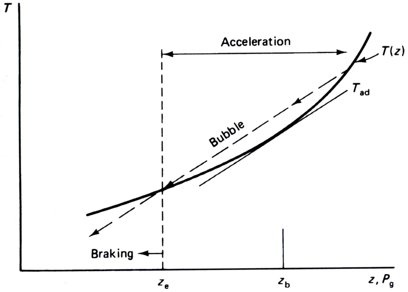
$$\frac{\mathrm{d}S}{\mathrm{d}r} > 0$$

(except when composition gradients are present).

Convection - Mixing Length Theory

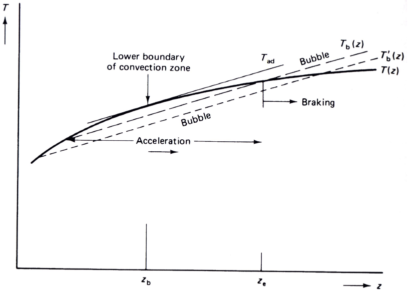
- In stellar evolution simulations, convection is treated using **mixing-length theory** (MLT).
- MLT uses a parameter, the *mixing length*, α_{MLT}, that describes how many pressure scale heights a bubble rises before mixing with its surroundings
- MLT is a strictly local theory, which makes it easy to implement, but has limitations
- For the sun, $\alpha_{\mathsf{MLT}} \approx 1.7$ seems to be a good approximation to reproduce the temperature structure of the outer convection zone, but better fits are obtained using $\alpha_{\mathsf{MLT}}(r)$.
- MLT does not do well describing non-local nature of convection and the behavior of convection at its boundaries.

Convective Overshooting



- If bubble move towards the boundary of convection they still have inertia and may not stop immediately
- accelerated in convection zone
- "braking" in overshoot region

Convective Penetration



A bubble coming from some distance (left) may have low enough entropy to still be accelerated outside the formal boundary of convection (z_b).

Summary on Local Stability

- Convection according to **Ledoux Criterion** when

$$\nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

- Convection according to **Schwarzschild Criterion** when

$$\nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}}$$

- **Semiconvection** when

$$\frac{\varphi}{\delta}\nabla_{\mu} > 0\,,\, \nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

- **Thermohaline convection** when

$$\frac{\varphi}{\delta}\nabla_{\mu} < 0\,,\, \nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

Summary on Convection

- In the stellar interior bubbles rise close to adiabatically, and the temperature gradient in the convection zone is close to adiabatic, but slightly steeper

$$\frac{\mathrm{d}S}{\mathrm{d}r} \lesssim 0$$

- The four temperature gradients in convection zone are in order of increasing steepness
  - adiabatic temperature gradient
  - temperature gradient of rising bubble (i.e., “up-flow”)
  - temperature gradient of surrounding media
  - (factious) *radiative* temperature gradient
- Convection zones are “well mixed” – close to chemically homogeneous