Astrophysics I: Stars and Stellar Evolution

AST 4001

Alexander Heger[1,2,3]

[1]School of Physics and Astronomy
University of Minnesota

[2]Theoretical Astrophysics Group, T-6
Los Alamos National Laboratory

[3]Department of Astronomy and Astrophysics
University of California at Santa Cruz

Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - Local Stability
   - Convection
2. Advanced Nuclear Burning Stages
   - Overview – Origin of the Elements
   - Helium Burning and Beyond

Summary on Local Stability

- Convection according to **Ledoux Criterion** when

$$\nabla_{\text{rad}} > \nabla_{\text{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

- Convection according to **Schwarzschild Criterion** when

$$\nabla_{\text{rad}} > \nabla_{\text{ad}}$$

- **Semiconvection** when

$$\frac{\varphi}{\delta}\nabla_{\mu} > 0\,,\, \nabla_{\text{rad}} < \nabla_{\text{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

- **Thermohaline convection** when

$$\frac{\varphi}{\delta}\nabla_{\mu} < 0\,,\, \nabla_{\text{rad}} < \nabla_{\text{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}$$

Summary on Convection

- In the stellar interior bubbles rise close to adiabatically, and the temperature gradient in the convection zone is close to adiabatic, but slightly steeper

$$\frac{\mathrm{d}S}{\mathrm{d}r} \lesssim 0$$

- The four temperature gradients in convection zone are in order of increasing steepness
    - adiabatic temperature gradient
    - temperature gradient of rising bubble (i.e., "up-flow")
    - temperature gradient of surrounding media
    - (factious) *radiative* temperature gradient
- Convection zones are "well mixed" – close to chemically homogeneous

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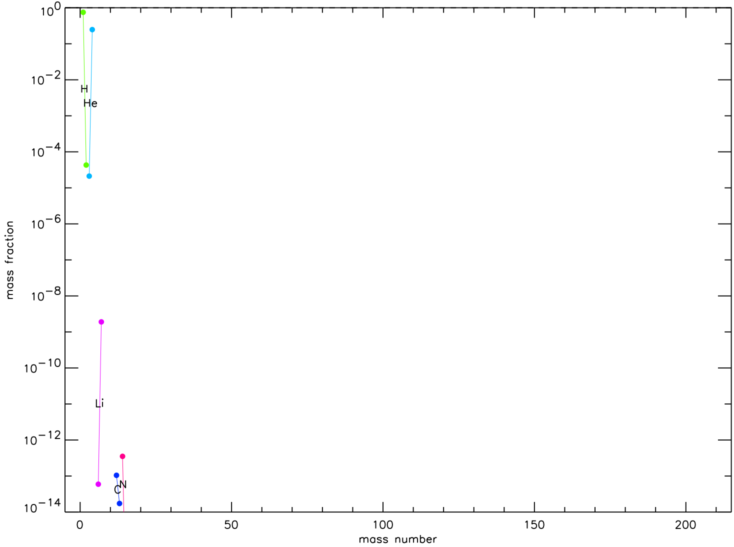
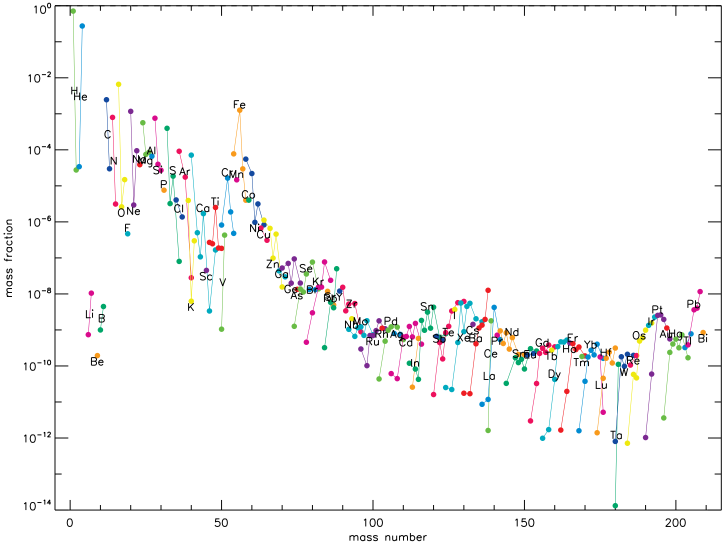
Composition of the Universe after the Big Bang

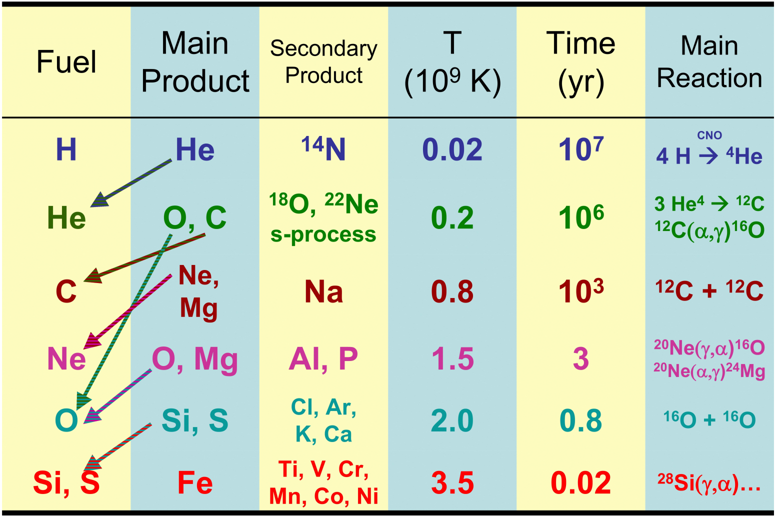


The Composition of the Sun



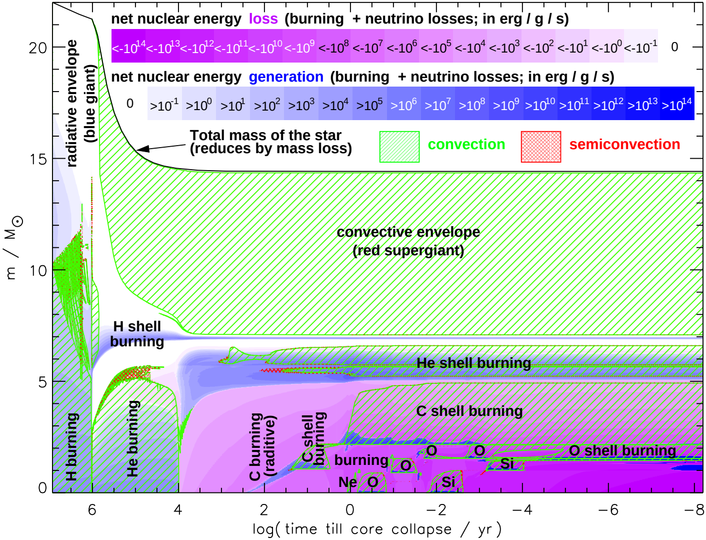
Overview - Burning Phases in Stars

$20\,M_\odot$ star



Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	**He**	**^{14}N**	**0.02**	**10^7**	**$4\,H \xrightarrow{CNO} {}^4He$**
He	**O, C**	**^{18}O, ^{22}Ne s-process**	**0.2**	**10^6**	**$3\,He^4 \rightarrow {}^{12}C$ $^{12}C(\alpha,\gamma)^{16}O$**
C	**Ne, Mg**	**Na**	**0.8**	**10^3**	**$^{12}C + {}^{12}C$**
Ne	**O, Mg**	**Al, P**	**1.5**	**3**	**$^{20}Ne(\gamma,\alpha)^{16}O$ $^{20}Ne(\alpha,\gamma)^{24}Mg$**
O	**Si, S**	**Cl, Ar, K, Ca**	**2.0**	**0.8**	**$^{16}O + {}^{16}O$**
Si, S	**Fe**	**Ti, V, Cr, Mn, Co, Ni**	**3.5**	**0.02**	**$^{28}Si(\gamma,\alpha)\ldots$**

Overview - Burning Phases in the Stellar Interior



Stellar Structure Equations - Nuclear Burning

stationary terms time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \tag{2}$$

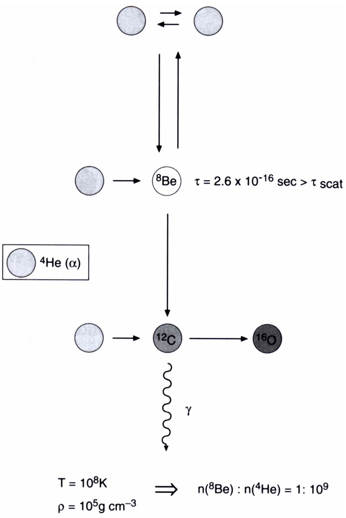
$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{3}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \tag{4}$$

$$\frac{\partial X_i}{\partial t} = f_i(\rho, T, \mathbf{X}) \tag{5}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

The Triple-α Reaction



Step 1:

$^4\text{He} + {}^4\text{He} \rightleftharpoons {}^8\text{Be}$

Built up equilibrium abundance of ^{8}Be

Lifetime of ^{8}Be is only 2.6×10^{-16} s!

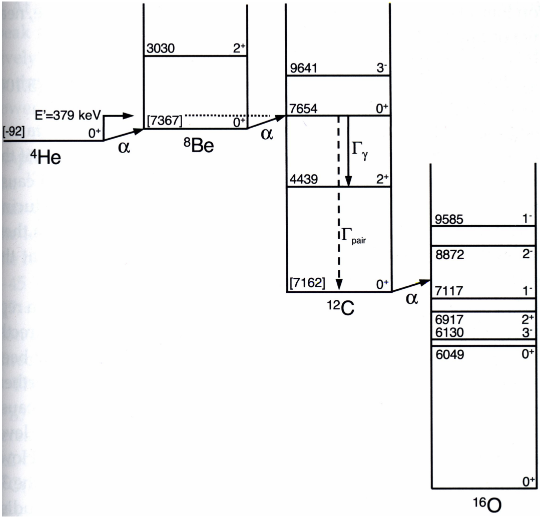
Step 2:

$^8\text{Be} + {}^4\text{He} \rightarrow {}^{12}\text{C} + \gamma$

$$Q_{3\alpha} = 7.275\,\text{MeV}$$

$$<\sigma v> \propto \rho^2 T^{40}$$

Helium Burning level scheme



Additional Reactions of Helium Burning

Oxygen Production

$^{4}\mathrm{He} + {}^{12}\mathrm{C} \rightarrow {}^{16}\mathrm{O} + \gamma$

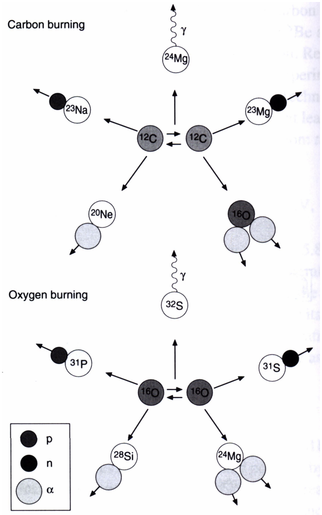
$Q = 7.162\,\mathrm{MeV}$

$\langle \sigma v \rangle \;\propto \rho T^{40}$

The final abundance of carbon is set by the competition of 3α and $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reactions;

The production of $^{16}\mathrm{O}$ can only start when a sufficient amount of $^{12}\mathrm{C}$ has been made.

Carbon and Oxygen Burning



Carbon Burning

$$\begin{aligned}
{}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{24}\mathrm{Mg} + \gamma \quad , & 13.931 \\
&\rightarrow {}^{23}\mathrm{Mg} + n \quad , & -2.605 \\
&\rightarrow {}^{23}\mathrm{Na} + p \quad , & 2.238 \\
&\rightarrow {}^{20}\mathrm{Ne} + \alpha \quad , & 4.616 \\
&\rightarrow {}^{16}\mathrm{O} + 2\alpha \quad , & -0.114
\end{aligned}$$

Average $Q = 13\,\mathrm{MeV}$

Oxygen Burning

$$\begin{aligned}
{}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\rightarrow {}^{32}\mathrm{S} + \gamma \quad , & 16.541 \\
&\rightarrow {}^{31}\mathrm{P} + p \quad , & 7.677 \\
&\rightarrow {}^{31}\mathrm{S} + n \quad , & 1.453 \\
&\rightarrow {}^{28}\mathrm{Si} + \alpha \quad , & 9.593 \\
&\rightarrow {}^{24}\mathrm{Mg} + 2\alpha \quad , & -0.393
\end{aligned}$$

Average $Q = 16\,\mathrm{MeV}$

Neon Burning

Neon burning proceeds by a combination of photo-disintegrations and α captures:

$$^{20}\text{Ne} + \gamma \rightarrow {}^{16}\text{O} + {}^{4}\text{He}\,, \quad Q = -4.73\,\text{MeV}$$

This reaction dominates over the inverse reaction known from helium burning for $T > 1.5 \times 10^9\,\text{K}$.

Subsequently, the ^{4}He is captured on another ^{20}Ne nucleus:

$$^{20}\text{Ne} + {}^{4}\text{He} \rightarrow {}^{24}\text{Mg} + \gamma.$$

The net result is

$$2\,{}^{20}\text{Ne} + \gamma \rightarrow {}^{16}\text{O} + {}^{24}\text{Mg} + \gamma\,, \quad Q = +4.583\,\text{MeV}$$

“Silicon” Burning

Actually, often we have more sulfur in the star than there is silicon, but it is custom to call this phase “silicon burning”.

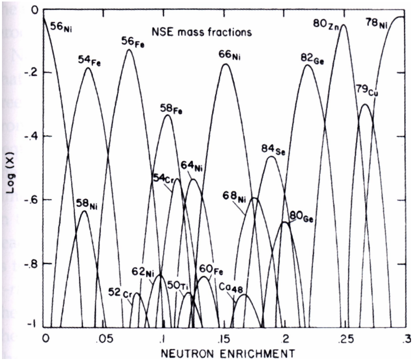
Typical burning temperature is $3 \ldots 3.5 \times 10^9\,\mathrm{K}$.

Similar to neon burning, silicon burning proceeds as a series of photo-disintegration reactions, mostly, (γ, α), and helium capture reactions, (α, γ) to build up iron group elements.

$$(\gamma, \alpha) \rightleftharpoons (\alpha, \gamma)$$

At the high T and ρ of these conditions, also *weak reactions* occur, converting protons into neutrons and leading to a *neutron excess*. This allows to actually make stable iron isotopes.

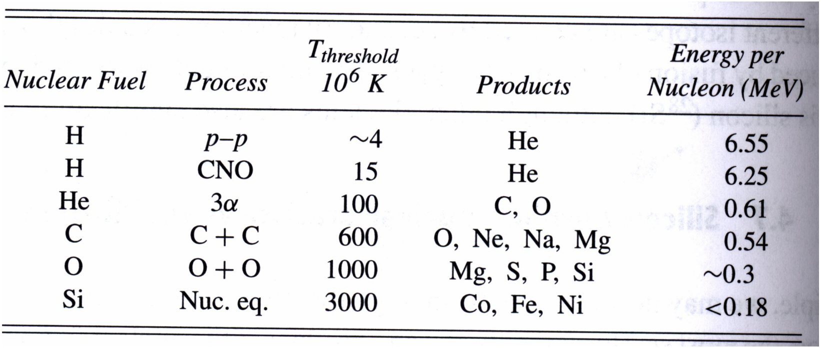
Beyond Silicon/Sulfur Burning



NSE distribution for
$T = 3.5 \times 10^9$ K,
$\rho = 10^7$ g/cm^3

After silicon burning T and ρ is so high that the nuclei are in **nuclear statistical equilibrium**, i.e., the reactions are fast compared to the evolution time-scale of the star, and the abundance distribution of the nuclei is given by a *Saha equation*.

Summary of Energetics



Nuclear Fuel	*Process*	$T_{threshold}$ 10^6 K	*Products*	*Energy per Nucleon (MeV)*
H	p–p	~4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C, O	0.61
C	C + C	600	O, Ne, Na, Mg	0.54
O	O + O	1000	Mg, S, P, Si	~0.3
Si	Nuc. eq.	3000	Co, Fe, Ni	<0.18