Astrophysics I: Stars and Stellar Evolution

AST 4001

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Overview

White Dwarf Mass-Radius Relation

- White dwarf stars: mass $\sim$ $M_\odot$, radius $\sim$ earth radius, cold $\Rightarrow$ Well described by (non-relativistic) degenerate equation of state with $\mu_e = 2$, $P_{e,deg} = K_1 \rho^{5/3} \Rightarrow K = K_1$ and $n = 1.5$.
- from the mass-radius relation,

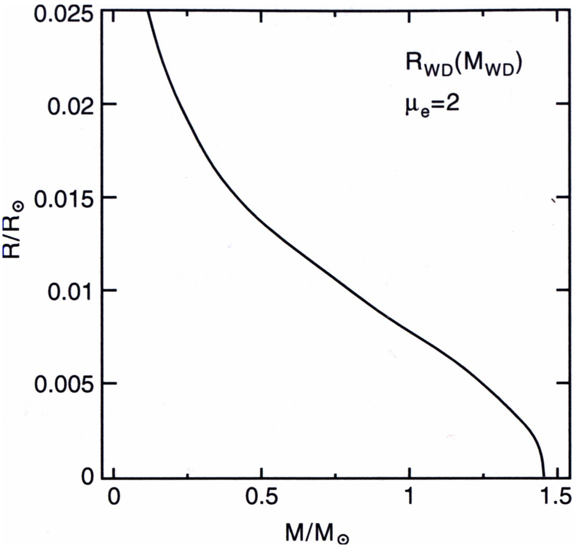
$$\left(\frac{GM}{M_n}\right)^{n-1}\left(\frac{R}{R_n}\right)^{3-n} = \frac{[(n+1)K]^n}{4\pi G}$$

we then find

$$R \propto M^{-1/3}, \quad \bar{\rho} \propto MR^{-3} \propto M^2$$

- NOTE: for increasing mass, the radius decreases and the density increases.
- eventually the density becomes so high that we can no longer use non-relativistic degenerate equation of state.

White Dwarf Mass-Radius Relation



- WD mass diverges for $M \longrightarrow 0$
- WD mass goes to zero at Chandrasekhar mass

White Dwarf Maximum Mass

- When we use the relativistic degenerate equation of state ($\mu_e = 2$),

$$P_{e,rel-deg} = \frac{hc}{8}\left(\frac{3}{\pi}\right)^{1/3}\frac{1}{u^{4/3}}\left(\frac{\rho}{\mu_e}\right)^{4/3} = K_2\rho^{4/3}$$

  we have a polytrope with $K = K_2$ and $n = 3$.
- we recall that for $n = 3$ there is only one unique mass as solution

$$M = 4\pi M_3\left(\frac{K}{\pi G}\right)^{3/2}$$

- This determines the maximum mass of white dwarfs

The Chandrasekhar Mass

- This limiting mass for degenerate stars is called the **Chandrasekhar Mass**

$$M_{\mathrm{Ch}} = \frac{M_3}{4\pi}\left(\frac{3}{2}\right)^{1/2}\left(\frac{hc}{G\mathrm{u}^{4/3}}\right)^{3/2}\mu_{\mathrm{e}}{}^{-2} = (5.836\,\mathrm{M}_\odot)\mu_{\mathrm{e}}{}^{-2}$$

$$M_{\mathrm{Ch}} = 1.459\,\mathrm{M}_\odot\left(\frac{\mu_{\mathrm{e}}}{2}\right)^{-2}$$

(Nobel Prize in Physics 1983)

- for an iron core with $\mu_{\mathrm{e}} = 2.15$ we obtain $M_{\mathrm{Ch}} = 1.26\,\mathrm{M}_\odot$
- for "hot" cores of massive stars partially degenerate relativistic equation of state has to be used
$\Rightarrow M_{\mathrm{crit}} > M_{\mathrm{Ch}}$

$$M_{\mathrm{crit}} \approx M_{\mathrm{Ch}}\left[1 + \frac{\pi^2 k^2 T^2}{\epsilon_{\mathrm{F}}{}^2}\right]$$

where ϵ_{F} is the Fermi energy for the relativistic and partially degenerate electrons, $Y_{\mathrm{e}} = 1/\mu_{\mathrm{e}}$,

$$\epsilon_{\mathrm{F}} = 1.11\left(\frac{\rho}{10^7\,\mathrm{g\,cm}^{-3}}Y_{\mathrm{e}}\right)^{1/3}\,\mathrm{MeV}$$

Overview

Eddington Limit

- The radiation pressure is given by $P_{\mathrm{rad}} = \frac{a}{3}T^4$, hence its gradient is

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{4a}{3}T^3\frac{\mathrm{d}T}{\mathrm{d}r}$$

- The radiative temperature gradient is given by

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\kappa L}{4acT^3 4\pi r^2}$$

where at the surface we use $F = L$, and we get

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{\kappa\rho L}{4\pi r^2 c}$$

Comparing this to the gravitational acceleration, (force per unit volume)

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho\frac{GM}{r^2}$$

- and solving for L we obtain

$$L_{\mathrm{edd}} = \frac{4\pi cGM}{\kappa}$$

Eddington Limit from Flux

- The outward flux at the surface is

$$H = \frac{L}{4\pi r^2}$$

- the force from radiation pressure is

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{\kappa\rho}{c}H = -\frac{\kappa\rho L}{c 4\pi r^2}$$

- setting these two equal, we again recover

$$L_{\mathrm{edd}} = \frac{4\pi c G M}{\kappa}$$

Eddington Limit - Approximations

- as simple approximation we often use just the electron scattering opacity, (with $\kappa_{\mathrm{es},0} = 0.4\,\mathrm{cm}^2\,\mathrm{g}^{-1}$)

$$\kappa = \kappa_{\mathrm{es}} = \frac{\kappa_{\mathrm{es},0}}{\mu_{\mathrm{e}}} \approx \frac{1}{2}\kappa_{\mathrm{es},0}(1+X)$$

- $\kappa_{\mathrm{es},0}$ is due to *Thompson scattering* on free electrons, with a cross section of $\sigma_{\mathrm{T}} = \left(\frac{8\pi}{3}\right)\left(\frac{e^2}{m_{\mathrm{e}}c^2}\right)^2 = 6.652\times10^{-25}\,\mathrm{cm}^2$; $\kappa_{\mathrm{es},0} = \sigma_{\mathrm{T}}/\mathrm{u}$
- for a fully ionized gas of pure hydrogen we hence have

$$L_{\mathrm{edd}} \approx \frac{4\pi cGM}{\kappa_{\mathrm{es},0}} = \frac{4\pi cGM\mathrm{u}}{\sigma_{\mathrm{T}}}$$

$$L_{\mathrm{edd}} \approx 1.3\times10^{38}\left(\frac{M}{\mathrm{M}_\odot}\right)\mathrm{erg\,s}^{-1} = 3.3\times10^{4}\left(\frac{M}{\mathrm{M}_\odot}\right)\mathrm{L}_\odot$$

Implications of Eddington Limit

- Maximum luminosity proportional to mass
  $\Rightarrow$ minimum lifetime of stars
  (assuming certain fraction of nuclear energy supply is being used)
- for "spherical" accretion this sets a maximum accretion rate from accretion luminosity
  (assuming radius of object or energy release efficiency by accretion)
  How fast can one assemble an astrophysical object?

Is Super-Eddington Luminosity Possible?

- Eddington Limit assumes strict spherical symmetry
  $\Rightarrow$ if problem not spherically symmetric, higher luminosity may be possible:
  - accretion disks
  - surface convection, turbulence.
  - "porosity" - radiation to escape between gas in regions of low opacity
  - "bubble" with high magnetic pressure and photon gas break out at the surface
- Eddington Limit assumes transport by radiation
  $\Rightarrow$ if energy is transported otherwise, higher L may be possible:
  - convection
  - sound waves
  - magnetic Alfvén waves

Beyond the Eddington Limit

- Eddington Limit assumes hydrostatic equilibrium
  $\Rightarrow$ in dynamic situations L can be higher
  - supernovae
  - gamma-ray burst
  - other kinds of transients
- Eddington Limit hydrogen gas and electron scattering opacity
  $\Rightarrow$ composition and state of gas can change limit
  - neutral hydrogen gas in red giant stars may have lower opacity
  - (pure) helium stars have fewer electrons per unit mass
  - metals may increase opacity at photosphere
- **Final Note:**
  for the Eddington limit we were interested in a global limit based on simple assumptions, that, e.g., is independent on radius

Eddington Quiz

Derive Eddington Luminosity for pure helium stars.

From small groups of 2-3 and write down your derivation.
You have two minutes.

Be prepared to present your group's solution on the black board.

Eddington Accretion Quiz

- Assume a star of radius R and mass M accretes material as "Eddington rate", i.e., the "accretion luminosity" equals the Eddington luminosity.
- For simplicity, assume that this accretion luminosity is just given by accretion rate and surface potential.
- Assume that all the energy that is released as the material hits the surface is radiated away.
- Assume that the gas is optically thin before it hits the surface, i.e., the gas does not "trap" the radiation.
- Assume pure hydrogen gas.

Compute this Eddington accretion rate.

From small groups of 2-3 and write down your derivation.
3 minutes.

Be prepared to present your group's solution on the black board.