Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

Eddington Limit - Approximations

- as simple approximation we often use just the electron scattering opacity, (with $\kappa_{\mathrm{es},0} = 0.4\,\mathrm{cm}^2\,\mathrm{g}^{-1}$)

$$\kappa = \kappa_{\mathrm{es}} = \frac{\kappa_{\mathrm{es},0}}{\mu_{\mathrm{e}}} \approx \frac{1}{2}\kappa_{\mathrm{es},0}(1+X)$$

- $\kappa_{\mathrm{es},0}$ is due to *Thompson scattering* on free electrons, with a cross section of $\sigma_{\mathrm{T}} = \left(\frac{8\pi}{3}\right)\left(\frac{e^2}{m_{\mathrm{e}}c^2}\right)^2 = 6.652\times10^{-25}\,\mathrm{cm}^2$; $\kappa_{\mathrm{es},0} = \sigma_{\mathrm{T}}/\mathrm{u}$
- for a fully ionized gas of pure hydrogen we hence have

$$L_{\mathrm{edd}} \approx \frac{4\pi cGM}{\kappa_{\mathrm{es},0}} = \frac{4\pi cGM\mathrm{u}}{\sigma_{\mathrm{T}}}$$

$$L_{\mathrm{edd}} \approx 1.3\times10^{38}\left(\frac{M}{\mathrm{M}_\odot}\right)\mathrm{erg\,s}^{-1} = 3.3\times10^{4}\left(\frac{M}{\mathrm{M}_\odot}\right)\mathrm{L}_\odot$$

Eddington Quiz

Derive Eddington Luminosity for pure helium stars.

A pure helium star has twice the Eddington luminosity of a star composed of pure hydrogen.

Eddington Accretion Quiz

- Assume a star of radius R and mass M accretes material as "Eddington rate", i.e., the "accretion luminosity" equals the Eddington luminosity.
- For simplicity, assume that this accretion luminosity is just given by accretion rate and surface potential.
- Assume that all the energy that is released as the material hits the surface is radiated away.
- Assume that the gas is optically thin before it hits the surface, i.e., the gas does not "trap" the radiation.
- Assume pure hydrogen gas.

Compute this *Eddington accretion rate.*

$$\frac{GM\dot{M}}{R} = \frac{4\pi cGM\mathrm{u}}{\sigma_\mathrm{T}} \quad \Rightarrow \quad \dot{M}_\mathrm{acc,Edd} = \frac{4\pi c\mathrm{u}}{\sigma_\mathrm{T}}R$$

Overview

Stellar Structure Equations

stationary terms | time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad (1)$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \quad (2)$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \quad (3)$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \quad (4)$$

$$\frac{\partial X_i}{\partial t} = f_i(\rho, T, \mathbf{X}) \quad (5)$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Eddington Model

- Using $P_{\rm rad} = \frac{1}{3}aT^4$ we can write

$$\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}P} = \left(\frac{\mathrm{d}r}{\mathrm{d}P}\right)\frac{\mathrm{d}\left(\frac{1}{3}aT^4\right)}{\mathrm{d}r} = \left(\frac{\mathrm{d}r}{\mathrm{d}P}\right)\frac{4}{3}aT^3\frac{\mathrm{d}T}{\mathrm{d}r}$$

$$\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}P} = \left(-\frac{r^2}{Gm\rho}\right)\left(\frac{4}{3}aT^3\right)\left(-\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{F}{4\pi r^2}\right) = \frac{F\kappa}{4\pi cGm}$$

- We can define a function η to describe the ratio of energy flow to enclosed mass, F/m as in terms of the total specific energy generation rate of the star

$$\frac{F}{m} = \eta\frac{L}{M}$$

- and obtain

$$\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}P} = \frac{L}{4\pi cGM}\kappa\eta$$

Eddington Model

- usually star burns most fuel in center (high F/m)
- usually opacity increases outward (high κ)
- $\Rightarrow$ Eddington makes simple assumption:

$$\kappa\eta = \text{constant} = \kappa_{\text{surf}}$$

where κ_{surf} is the surface opacity ($\eta = 1$)

- we now have

$$\frac{\mathrm{d}P_{\text{rad}}}{\mathrm{d}P} = \frac{L\kappa_{\text{surf}}}{4\pi cGM} = \text{constant}$$

and obtain

$$P_{\text{rad}} = \frac{L\kappa_{\text{surf}}}{4\pi cGM}P$$

- $\Rightarrow$ constant ratio of gas pressure to total pressure

Eddington Model

- We recall

$$\beta = \frac{P_{\text{gas}}}{P}$$

- hence

$$P_{\text{rad}} = P - P_{\text{gas}} = (1 - \beta)P$$

- and we can write

$$L = \frac{4\pi c G M}{\kappa_{\text{surf}}}(1 - \beta) = L^*_{\text{edd}}(1 - \beta)$$

  where L^*_{edd} is a variation of Eddington luminosity considering total surface opacity, in this simplified model.

- This implies that the luminosity reaches Eddington luminosity for a star dominated by radiation pressure ($\beta \to 0$)

Eddington Model

- We may write the pressure in the form

$$P = \frac{P_{\text{rad}}}{1-\beta} = \frac{a}{3}T^4\frac{1}{1-\beta}$$

  or in the form

$$P = \frac{P_{\text{gas}}}{\beta} = \frac{\mathcal{R}T\rho}{\mu}\frac{1}{\beta}$$

- combining these two and solving for T we obtain:

$$T = \left[\frac{3\mathcal{R}(1-\beta)}{a\mu\beta}\right]^{1/3}\rho^{1/3}$$

- using again $P = \mathcal{R}T\rho/\mu\beta$ we can now write the equation of state (EOS) in the form

$$P = K\rho^{4/3}\,, \quad K = \left[\frac{3\mathcal{R}^4(1-\beta)}{a\mu^4\beta^4}\right]^{1/3}$$

- $\Rightarrow$ polytrope of index $n = 3$

Eddington Quadratic Equation

- Recall that for $n = 3$ we have a unique relation between K and mass

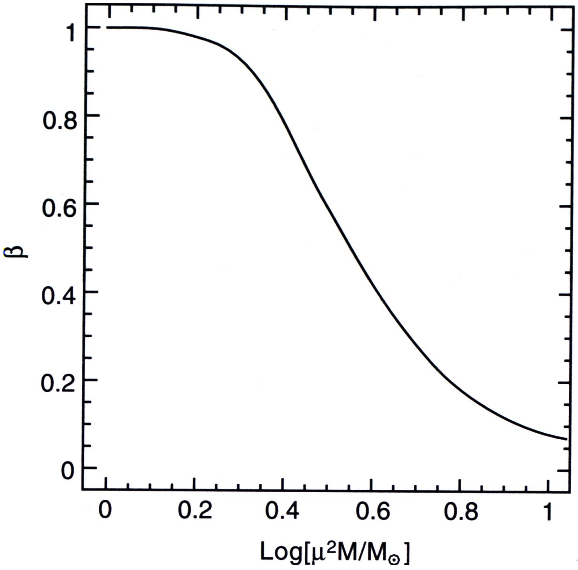
$$M = 4\pi M_3 \left(\frac{K}{\pi G}\right)^{3/2}$$

- NOTE: for completely degenerate stars, we used that to derive the Chandrasekhar mass, as K was derived from elementary physics, the degenerate EOS.
- but now this gives a relation between that allows to compute β for a given M and μ (function of given gas composition):

$$1 - \beta = 0.003 \left(\frac{M}{\mathrm{M}_\odot}\right)^2 \mu^4 \beta^4$$

- It is called the *Eddington Quadratic Equation*
- This usually gives a good approximation for non-degenerate main-sequence stars.

Eddington Quadratic Equation



normalized solution for the Eddington quadratic equation

Properties Eddington Model

- for given composition (fixed μ), β decreases as M increases
- inserting the solution function $\beta(M,\mu)$ into $L = L^*_{\mathrm{edd}}(1-\beta)$ we obtain

$$L = 0.003\,\frac{4\pi c G \mathrm{M}_\odot}{\kappa_{\mathrm{surf}}}\mu^4\beta(M,\mu)^4\left(\frac{M}{\mathrm{M}_\odot}\right)^3$$

- recover mass-luminosity relation
- as star evolves, μ increases, hence it gets closer to Eddington limit and its luminosity rises;
  but unless the star is well mixed (e.g., fully convective), there will no longer be a uniform μ throughout the star

M-L Quiz

For $M \to \infty$, recover the relation $L \propto M$ from

$$L = 0.003 \, \frac{4\pi c G \mathrm{M}_\odot}{\kappa_{\mathrm{surf}}} \mu^4 \beta(M,\mu)^4 \left(\frac{M}{\mathrm{M}_\odot}\right)^3$$

and

$$1 - \beta = 0.003 \left(\frac{M}{\mathrm{M}_\odot}\right)^2 \mu^4 \beta^4$$

From small groups of 2-3 and write down your derivation.
3 minutes.

Be prepared to present your group's solution on the black board.

M-L Quiz Solution

For $M \to \infty$, recover the relation $L \propto M$ from

$$L = 0.003\,\frac{4\pi c G \mathrm{M}_\odot}{\kappa_{\mathrm{surf}}}\mu^4\beta(M,\mu)^4\left(\frac{M}{\mathrm{M}_\odot}\right)^3$$

and

$$1-\beta = 0.003\left(\frac{M}{\mathrm{M}_\odot}\right)^2\mu^4\beta^4$$

In the second equation, the right hand side is finate $(0<\beta<1)$ hence as $M\to\infty$ on the right hand side $\beta\to 0$ is required. This menas that on the left hand side we can neglect β, and we have $\beta^4 \propto M^{-2}$. If we put this into the first equation, we are left with only one power in M, hence $L \propto M$.

Overview

1. Recap
   - Eddington Limit
2. Simple Stellar Models
   - Stellar Structure Equations
   - Standard Stellar Model
   - Final Quiz on Eddington
3. Next Class
   - Computer Lab

Computer Lab

- Class tomorrow, 10:10-11:00, **Walter Library, room 575**
  Meet at reception on 5th floor *on time*
  (class room is in secured area)
- **Be prepared.**
- have a look at **WIKI** on web bage
  use this to report your experince, post questions.
- Unix introduction
  http://static.msi.umn.edu/tutorial/hardwareprogramming/intro_to_unix_06_07_06.pdf
- emacs introduction
  http://www.gnu.org/software/emacs/manual/emacs.html
- FORTRAN introduction
  http://www.cs.mtu.edu/ shene/COURSES/cs201/NOTES/intro.html