Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

Thin Shell Instability

- The mass of the shell is $\Delta m \approx 4\pi r_0^2 l\rho$, $(l = r - r_0)$ and therefore we have for the density

$$\frac{\mathrm{d}\rho}{\rho} = -\frac{\mathrm{d}l}{l} = -\frac{\mathrm{d}r}{l} = -\frac{\mathrm{d}r}{r}\frac{r}{l}$$

- in hydrostatic equilibrium, the pressure in the shell depends on the layers above and varies as r^{-4}:

$$\frac{\mathrm{d}P}{P} = -4\frac{\mathrm{d}r}{r} = 4\frac{l}{r}\frac{\mathrm{d}\rho}{\rho}$$

- using the general EOS we obtain

$$\left(4\frac{l}{r} - a\right)\frac{\mathrm{d}\rho}{\rho} = b\frac{\mathrm{d}T}{T}$$

- since $b > 0$, to have $\rho\downarrow\rightarrow T\downarrow$ we require $4l/r > a$
- For a thin shell $l/r \rightarrow 0$, hence $\rho\downarrow\rightarrow T\uparrow\Rightarrow$ instability!

Stability from the EOS

- simple equations of state:
  - ideal gas: $\gamma_{\mathrm{ad}} = 5/3 \Rightarrow$ stability
  - non-relativistic degenerate gas: $\gamma_{\mathrm{ad}} = 5/3 \Rightarrow$ stability
  - relativistic degenerate gas: $\gamma_{\mathrm{ad}} = 4/3 \Rightarrow$ neutral stability
  - pure radiation gas: $\gamma_{\mathrm{ad}} = 4/3 \Rightarrow$ neutral stability
- ideal gas with radiation

$$\gamma_{\mathrm{ad}} = \frac{5\beta^2 + 8(1-\beta)(4+\beta)}{3\beta^2 + 6(1-\beta)(4+\beta)}$$

  for $\beta \rightarrow 0$ we obtain $\gamma_{\mathrm{ad}} \rightarrow 4/3$ (radiation dominated)
- ionization: γ_{ad} can drop below $4/3$
- electron-positron pair creation, iron and helium disintegration: γ_{ad} can drop below $4/3$
- general relativity: critical value of $\gamma_{\mathrm{ad}} > 4/3$.

Dynamical Stability

Summary

- It can be shown that if $\gamma_{\mathrm{ad}} > 4/3$ everywhere in the star, it is dynamically stable
- It is neutrally stable if $\gamma_{\mathrm{ad}} = 4/3$ everywhere in the star
- global dynamical *instability* of the star results if

$$\langle\gamma_{\mathrm{ad}}\rangle_{\frac{P}{\rho}} \equiv \frac{\int_0^M \gamma_{\mathrm{ad}} \frac{P}{\rho}\,\mathrm{d}m}{\int_0^M \frac{P}{\rho}\,\mathrm{d}m} < \frac{4}{3}$$

Overview

1. Recap
   - Thin Shell Instabilility
   - Cases of Dynamical (In)Stability
   - Dynamical Stability
2. Stellar Evolution
   - Regimes of the Temperature-Density Plane
   - Regimes of Nuclear Burning
   - Regimes of Stellar Evolution
3. Next Class
   - Computer Lab

Deviding line Between Ideal Gas and NR Deg. Gas

- ideal gas pressure

$$P = \frac{\mathcal{R}}{\mu}\rho T = K_0 \rho T$$

$\Rightarrow$

$$\log P = \log K_0 + \log \rho + \log T$$

- (non-rel.) degenerate gas

$$P = K_1 \rho^{5/3}$$

$\Rightarrow$

$$\log P = \log K_1 + \frac{5}{3} \log \rho$$

- the location where both pressure contributions become the same is defined by

$$\log \rho = \frac{3}{2} \log T + \text{const.}$$

Deviding line Between Ideal Gas and Rel. Deg. Gas

- ideal gas pressure

$$P = \frac{\mathcal{R}}{\mu}\rho T = K_0 \rho T$$

$\Rightarrow$

$$\log P = \log K_0 + \log \rho + \log T$$

- relativistic degenerate gas

$$P = K_1 \rho^{4/3}$$

$\Rightarrow$

$$\log P = \log K_2 + \frac{4}{3} \log \rho$$

- the location where both pressure contributions become the same is defined by

$$\log \rho = 3 \log T + \text{const.}$$

Deviding line Between Rel. and Non-Rel. Degenerate Gas

- non-rel. degenerate gas

$$P = K_1 \rho^{5/3}$$

$\Rightarrow$

$$\log P = \log K_1 + \frac{5}{3} \log \rho$$

relativistic degenerate gas

$$P = K_2 \rho^{4/3}$$

$\Rightarrow$

$$\log P = \log K_2 + \frac{4}{3} \log \rho$$

- the location where both pressure contributions become the same is defined by

$$\log \rho = 3 \log \left(\frac{K_2}{K_1} \right) = \text{const.}$$

Deviding line Between Ideal Gas and Radiation Pressure

- ideal gas pressure

$$P = \frac{\mathcal{R}}{\mu}\rho T = K_0 \rho T$$

$\Rightarrow$

$$\log P = \log K_0 + \log \rho + \log T$$

- radiaiton pressure

$$P = \frac{a}{3} T^4$$

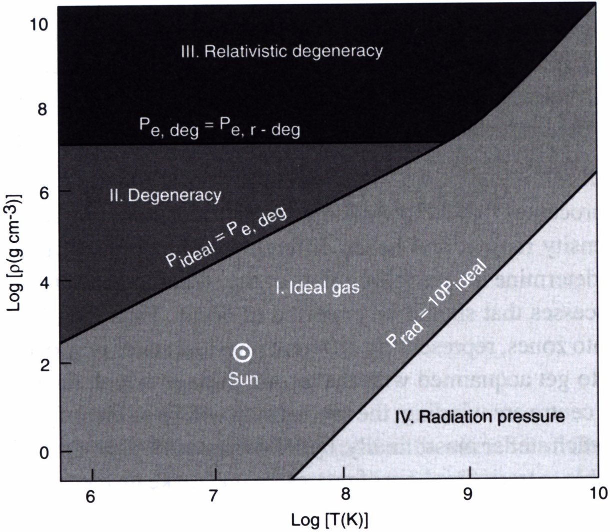
$\Rightarrow$

$$\log P = \log\left(\frac{a}{3}\right) + 4 \log T$$

- the location where both pressure contributions become the same is defined by

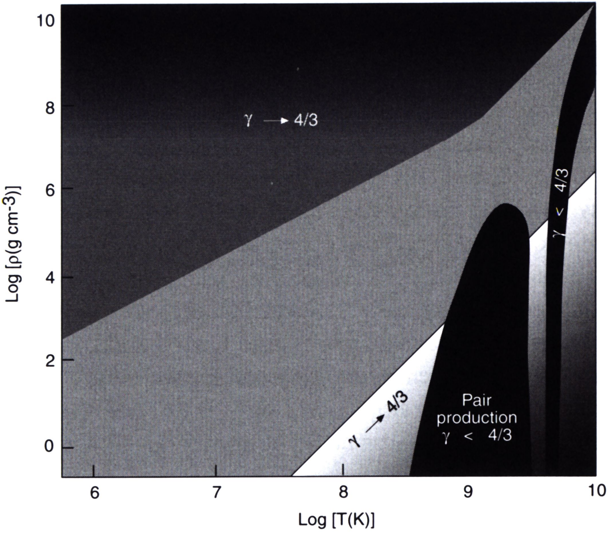
$$\log \rho = 3 \log T + \text{const.}$$

Regimes of the Equation of State



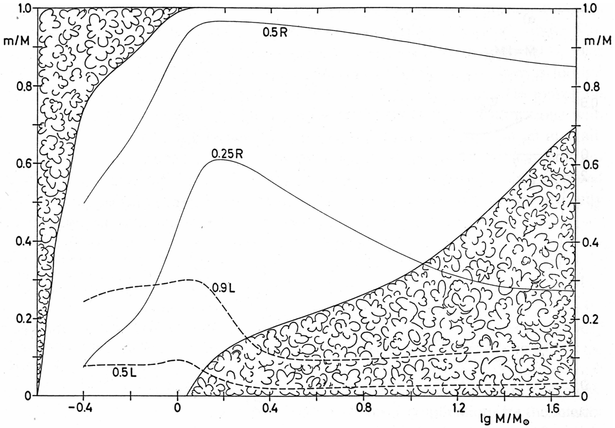
Equation of state in the temperature-density diagram

Regimes of Stability



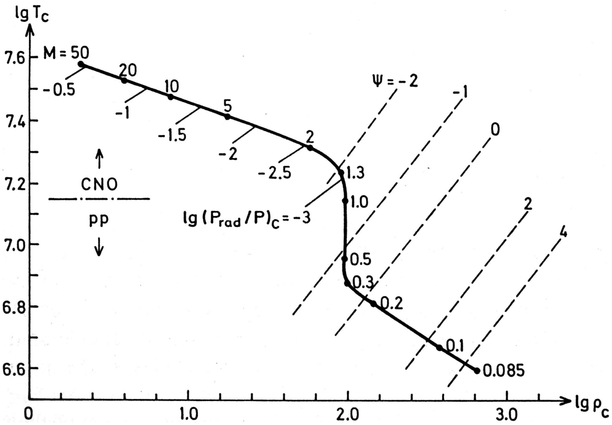
Regimes of *dynamic stability* in the temperature-density diagram

Regimes of Convection (local [in]stability)



Regimes of convection as a function of mass (x-axis) and fractional stellar mass (y-axis) on the **Z**ero-**A**ge **M**ain **S**equence (ZAMS).

Regimes of the EOS for Main-Sequence Stars



Equation of state in the density-temperature diagram for main sequence stars.

(note reversal of T and ρ)

Burning Phases in Stars

20 $M_\odot$ star

Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	He	^{14}N	0.02	10^7	$4\,H \xrightarrow{CNO} {}^4He$
He	O, C	^{18}O, ^{22}Ne s-process	0.2	10^6	$3\,He^4 \rightarrow {}^{12}C$ $^{12}C(\alpha,\gamma)^{16}O$
C	Ne, Mg	Na	0.8	10^3	$^{12}C + {}^{12}C$
Ne	O, Mg	Al, P	1.5	3	$^{20}Ne(\gamma,\alpha)^{16}O$ $^{20}Ne(\alpha,\gamma)^{24}Mg$
O	Si, S	Cl, Ar, K, Ca	2.0	0.8	$^{16}O + {}^{16}O$
Si, S	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	$^{28}Si(\gamma,\alpha)\ldots$

Regimes of Nuclear Burning

- assume *arbitrary* minimum energy generation rate for burning to becom important, say $q_{\mathrm{min}} \approx 10^3\,\mathrm{erg\,g^{-1}\,s^{-1}}$
- assume general power-law for energy generation rate

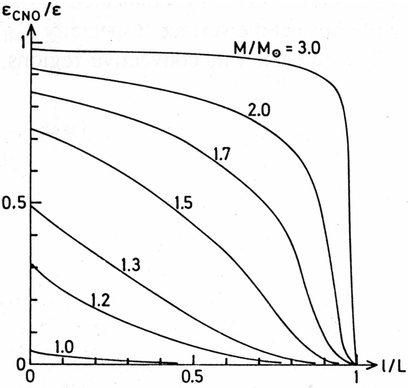
$$q = q_0 \rho^m T^n$$

- q rises above q_{min} for

$$\log \rho = -\frac{m}{n} \log T + \frac{1}{m} \log \left(\frac{q_{\mathrm{min}}}{q} \right)$$

- In reality, $n = n(T)$
  $\Rightarrow$ not straight lines but bent
- hydrogen burning has different contributions (pp chains, CNO cycle)
- helium burning has contributions from 3α and $^{12}\mathrm{C}(\alpha,\gamma)$

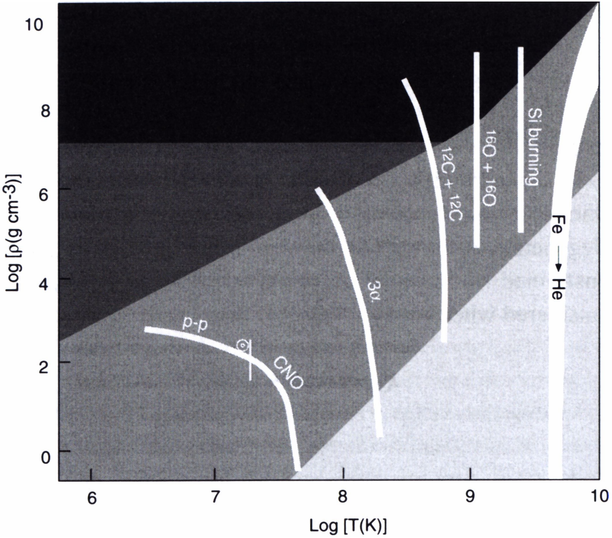
PP and CNO Cycle Competition



Fraction of the energy generation by the CNO cycle during hydrogen burning on the main sequence for different stellar masses as a function of the integrated stellar luminosity "l" as a radial coordinate, normalized to the total luminosity L of the star.

$$F(m) = l(m) = \int_0^m \epsilon(m')\mathrm{d}m'$$

Regimes of Burning



Regimes of burning in the temperature-density diagram

Regimes of Stellar Evolution

- Recall

$$P_\mathrm{c} = \sqrt[3]{4\pi} B_n G M^{2/3} \rho_c^{4/3}$$

- for ideal gas, $P_\mathrm{c} = K_0 \rho_\mathrm{c} T_\mathrm{c}$ and we obtain

$$\rho_\mathrm{c} = \frac{K_0^3}{4\pi B_n^3 G^3} \frac{T_\mathrm{c}^3}{M^2}$$

$$\Rightarrow \log \rho_\mathrm{c} = 3 \log T - 2 \log M + \mathrm{const.}$$

- for non-rel. degenerate gas $P_\mathrm{c} = K_1 \rho_\mathrm{c}^{5/3}$ we obtain

$$\rho_\mathrm{c} = 4\pi \left(\frac{B_{1.5} G}{K_1} \right)^3 M^2$$

$\Rightarrow$ parallel lines at $\log \rho_\mathrm{c} = 2 \log M + \mathrm{const.}$

Quiz

Find a relation for relativistic degenerate gas.

- Work and discuss in groups of 2-3.
- 3 min
- Please write up your solution.
- Please sign with your names and to hand in.
- (no grades)

Quiz Solution

Find a relation for relativistic degenerate gas.

for rel. degenerate (electron) gas

$$P_\mathrm{c} = K_2 \rho_\mathrm{c}^{4/3}$$

in

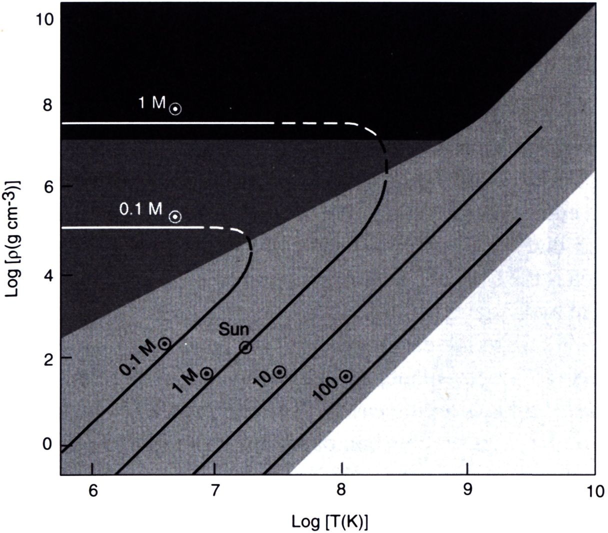
$$P_\mathrm{c} = \sqrt[3]{4\pi} B_n G M^{2/3} \rho_c^{4/3}$$

we obtain (using $M_3 = (4B_3)^{-3/2}$)

$$M = \frac{1}{\sqrt{4\pi}} \left(\frac{K_2}{G B_3} \right)^{3/2} = 4\pi M_3 \left(\frac{K_2}{\pi G} \right)^{3/2}$$

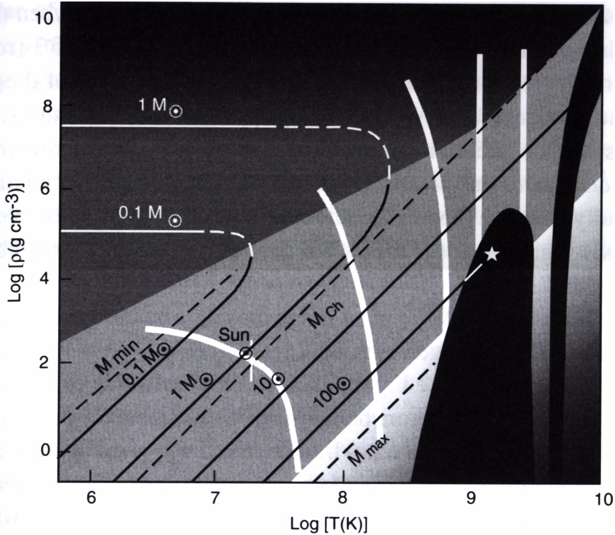
...the Chandrasekar Mass!

Domains of Stellar Mass



Regimes of stellar mass in the temperature-density diagram

Evolution Tracks



Evolution of Stars in the temperature-density diagram

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Computer Lab

- Class tomorrow, 10:10-11:00, **Walter Library, room 575**
  Meet at reception on 5th floor *on time*
  (class room is in secured area)
- try to familiarize yourself with IDL (use physics computers)
- have a look at **WIKI** on web bage
  use this to report your experince, post questions.
- Unix introduction
  http://static.msi.umn.edu/tutorial/hardwareprogramming/intro_to_unix_06_07_06.pdf
- emacs introduction
  http://www.gnu.org/software/emacs/manual/emacs.html
- FORTRAN introduction
  http://www.cs.mtu.edu/ shene/COURSES/cs201/NOTES/intro.html