Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

Regimes of Stellar Evolution

- Recall

$$P_c = \sqrt[3]{4\pi} B_n G M^{2/3} \rho_c^{4/3}$$

- for ideal gas, $P_c = K_0 \rho_c T_c$ and we obtain

$$\rho_c = \frac{K_0^3}{4\pi B_n^3 G^3} \frac{T_c^3}{M^2}$$

$\Rightarrow$ parallel lines with $\log \rho_c = 3 \log T - 2 \log M + \text{const.}$

- for non-rel. degenerate gas $P_c = K_1 \rho_c^{5/3}$ we obtain

$$\rho_c = 4\pi \left(\frac{B_{1.5} G}{K_1} \right)^3 M^2$$

$\Rightarrow$ parallel lines at $\log \rho_c = 2 \log M + \text{const.}$

Quiz Solution

Find a relation for relativistic degenerate gas.

for rel. degenerate (electron) gas

$$P_\mathrm{c} = K_2 \rho_\mathrm{c}^{4/3}$$

in

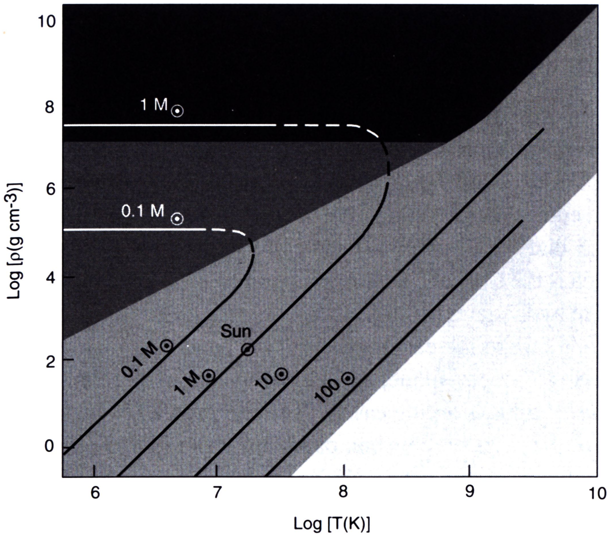
$$P_\mathrm{c} = \sqrt[3]{4\pi} B_n G M^{2/3} \rho_c^{4/3}$$

we obtain (using $M_3 = (4B_3)^{-3/2}$)

$$M = \frac{1}{\sqrt{4\pi}} \left(\frac{K_2}{G B_3} \right)^{3/2} = 4\pi M_3 \left(\frac{K_2}{\pi G} \right)^{3/2}$$

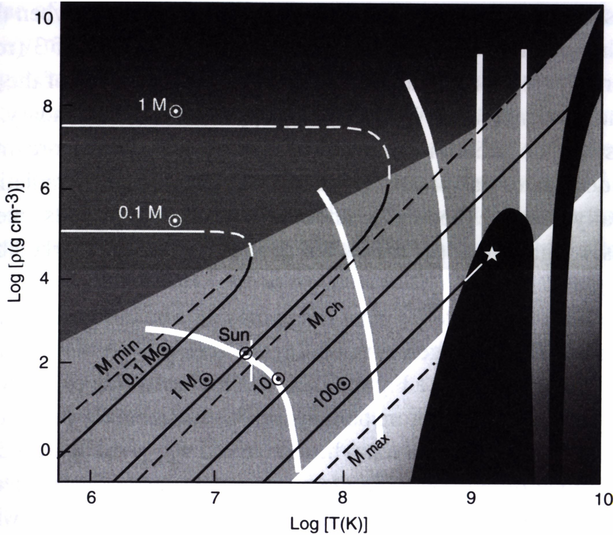
...the Chandrasekar Mass!

Domains of Stellar Mass



Regimes of stellar mass in the temperature-density diagram

Evolution Tracks



Evolution of Stars in the temperature-density diagram

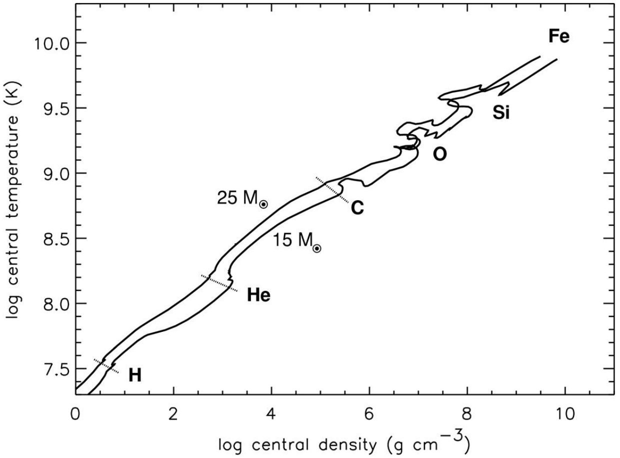
Overview

Burning Phases in Stars

20 $M_\odot$ star

Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	He	^{14}N	0.02	10^7	$4\,H \xrightarrow{CNO} {}^4He$
He	O, C	^{18}O, ^{22}Ne s-process	0.2	10^6	$3\,He^4 \rightarrow {}^{12}C$ $^{12}C(\alpha,\gamma)^{16}O$
C	Ne, Mg	Na	0.8	10^3	$^{12}C + {}^{12}C$
Ne	O, Mg	Al, P	1.5	3	$^{20}Ne(\gamma,\alpha)^{16}O$ $^{20}Ne(\alpha,\gamma)^{24}Mg$
O	Si, S	Cl, Ar, K, Ca	2.0	0.8	$^{16}O + {}^{16}O$
Si, S	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	$^{28}Si(\gamma,\alpha)\ldots$

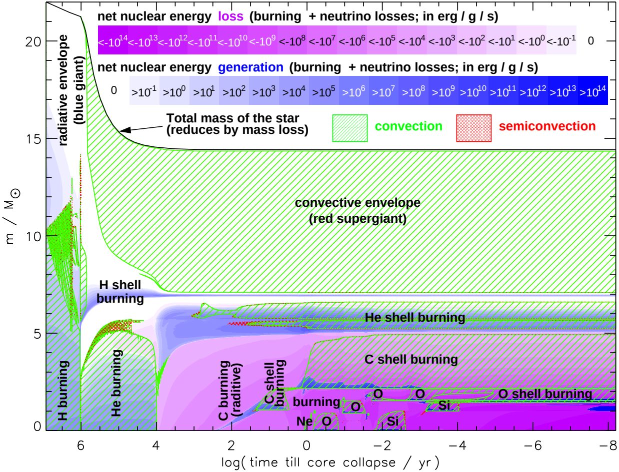
Evolution of Stars, 15 $M_\odot$ and 25 $M_\odot$



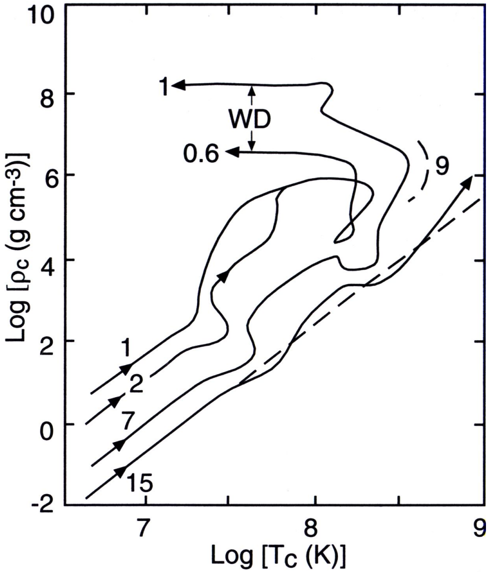
Evolution of central temperature and density for initial stellar masses of 15 $M_\odot$ and 25 $M_\odot$ in the density-temperature diagram

(note reversal of T and ρ)

Burning Phases in the Stellar Interior

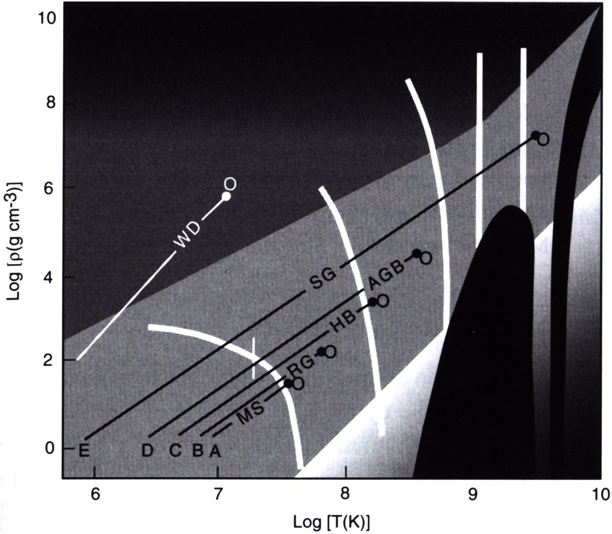


Evolution of Stars, 1–15 $M_\odot$



Evolution of central temperature and density for initial stellar masses of $1\,M_\odot$ to $15\,M_\odot$ in the temperature-density diagram.

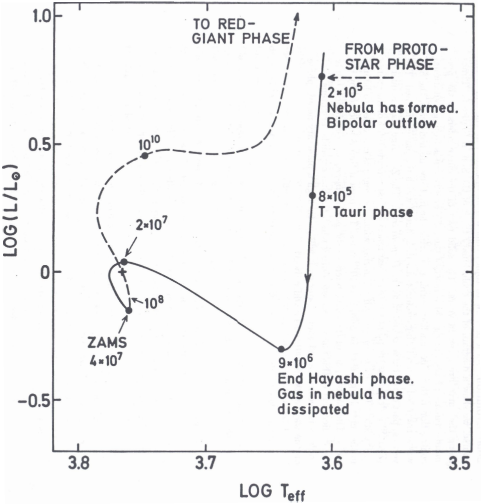
Configuration of a 10 $M_\odot$ Star



Configuration of a 10 $M_\odot$ star at different evolution phases in the temperature-density diagram.

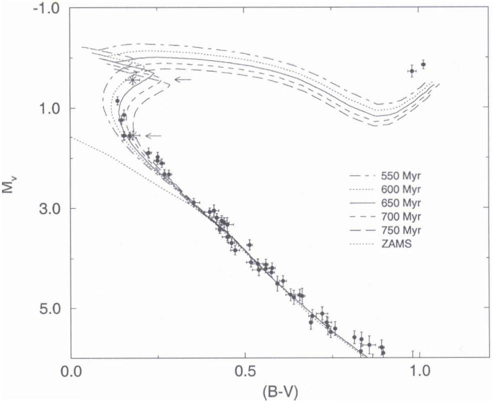
Overview

Evolution of the Sun in the HRD



Evolution of the sun from formation through hydrogen burning

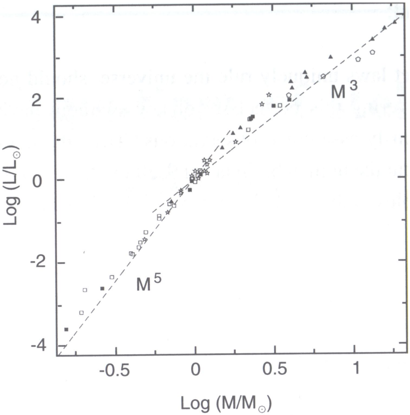
Main Sequance in a Star Cluster



Hyades cluster and stellar tracks

$$\log L = \alpha \log T_{\rm eff} + {\rm const.}$$

Mass-Luminosity Relation



Mass-Luminosity relation for (zero-age) main-sequence (ZAMS) stars

$$L \propto M^{\nu}$$

with $\nu = 3 \ldots 5$.
Can be calibrated piecewise to

$$\frac{L}{\mathrm{L}_\odot} = \left(\frac{M}{\mathrm{M}_\odot}\right)^{\nu}$$

Stellar Structure Equations

stationary terms time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad (1)$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \quad (2)$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \quad (3)$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \quad (4)$$

$$\frac{\partial X_i}{\partial t} = f_i(\rho, T, \mathbf{X}) \quad (5)$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Simplified Stellar Structure Equations

- only radiative temperature gradient: $\nabla = \nabla_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa FP}{mT^4}$
- only simple law for nuclear burning: $\varepsilon = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu = q_0 \rho T^n$
- only ideal gas pressure: $P = P_{\mathrm{gas}} = \frac{\mathcal{R}T\rho}{\mu}$

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

$$\frac{\partial T}{\partial m} = -\frac{3}{4ac}\frac{\kappa}{T^3}\frac{F}{(4\pi r^2)^2}$$

$$\frac{\partial F}{\partial m} = q_0 \rho T^n$$

$$P = \frac{\mathcal{R}T\rho}{\mu}$$

Dimensionless Stellar Structure Equations

- all functions $r(m)$, $P(m)$, $\rho(m)$, $T(m)$, and $F(m)$ need to be solved in range $0 \leq m \leq M$
- free parameter: mass M
- parameters κ, q_0, μ, and n determined from physics
- introduce dimension-less variable

$$x = \frac{m}{M}$$

- we can now write a set of dimension-less equations with functions $f_i(x)$ for these 5 quantities:

  $r = f_1(x)R_*$
  $P = f_2(x)P_*$
  $\rho = f_3(x)\rho_*$
  $T = f_4(x)T_*$
  $F = f_5(x)F_*$

Dimensionless Stellar Structure Equations

- substituting
  $m = Mx$,
  $r = f_1(x)R_*$,
  $P = f_2(x)P_*$ into

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

  we obtain

$$\frac{P_*}{M}\frac{\mathrm{d}f_2}{\mathrm{d}x} = -\frac{GMx}{4\pi f_1^4 R_*^4}$$

- If we define

$$P_* = \frac{GM^2}{R_*^4}$$

  we may write

$$\frac{\mathrm{d}f_2}{\mathrm{d}x} = -\frac{x}{4\pi f_1^4}$$

Quiz

Try this for the other four equations

- Work and discuss in groups of 2-3.
- 3 min
- Please write up your solution.
- Please sign with your names and to hand in.
- (no grades)

Quiz - equations

Try this for the other four equations

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$
$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$
$$\frac{\partial T}{\partial m} = -\frac{3}{4ac}\frac{\kappa}{T^3}\frac{F}{(4\pi r^2)^2}$$
$$\frac{\partial F}{\partial m} = q_0 \rho T^n$$
$$P = \frac{\mathcal{R} T \rho}{\mu}$$

$$r = f_1(x) R_*$$
$$P = f_2(x) P_*$$
$$\rho = f_3(x) \rho_*$$
$$T = f_4(x) T_*$$
$$F = f_5(x) F_*$$

Dimensionless Stellar Structure Equations

In a similar way we can re-write the entire set

$$\frac{\mathrm{d}f_2}{\mathrm{d}x} = -\frac{x}{4\pi f_1^4} \quad , \quad P_* = \frac{GM^2}{R_*^4}$$

$$\frac{\mathrm{d}f_1}{\mathrm{d}x} = \frac{1}{4\pi f_1^2 f_3} \quad , \quad \rho_* = \frac{M}{R_*^3}$$

$$f_2 = f_3 f_4 \quad , \quad T_* = \frac{\mu P_*}{\mathcal{R}\rho_*}$$

$$\frac{\mathrm{d}f_4}{\mathrm{d}x} = -\frac{3f_5}{4f_4^3\left(4\pi f_1^2\right)^2} \quad , \quad F_* = \frac{ac}{\kappa}\frac{T_*^4 R_*^4}{M}$$

$$\frac{\mathrm{d}f_5}{\mathrm{d}x} = f_3 f_4^n \quad , \quad F_* = q_0 \rho_* T_*^n M$$

$\Rightarrow$ *homology* of solution as function of M!

Dimensionless Stellar Structure Equations

- substituting $P_* = \frac{GM^2}{R_*^4}$ and $\rho_* = \frac{M}{R_*^3}$ into $T_* = \frac{\mu P_*}{\mathcal{R}\rho_*}$ we obtain:

$$T_* = \frac{\mu G}{\mathcal{R}} \frac{M}{R_*}$$

- adding this into $F_* = \frac{ac}{\kappa} \frac{T_*^4 R_*^4}{M}$ we obtain

$$F_* = \frac{ac}{\kappa} \left(\frac{\mu G}{\mathcal{R}}\right)^4 M^3$$

- we recover $L \propto M^3$
- $\tau_{\rm MS} = \frac{M}{L} \propto M^{-2}$
- but this relation also holds for any value of x - inside star at same relative mass coordinate

Dimensionless Stellar Structure Equations

- substituting $F_* = \frac{ac}{\kappa}\left(\frac{\mu G}{\mathcal{R}}\right)^4 M^3$ into $F_* = q_0 \rho_* T_*^n M$ and using $P_* = \frac{GM^2}{R_*^4}$, $\rho_* = \frac{M}{R_*^3}$, and $T_* = \frac{\mu P_*}{\mathcal{R}\rho_*}$ we obtain:

$$R_* \propto M^{\frac{n-1}{n+3}}$$

- $\Rightarrow$ for large n (CNO cycle: $n \approx 14 \ldots 16$):
  roughly $R_* \propto M$
  (big stars)
- $\Rightarrow$ for small n (pp chains: $n = 4$):
  $R \propto M^{3/7}$

Dimensionless Stellar Structure Equations

- inserting $R_* \propto M^{\frac{n-1}{n+3}}$ into $\rho_* = \frac{M}{R_*^3}$ we obtain

$$\rho_* \propto M^{2\frac{3-n}{3+n}}$$

- since $n > 3$:
  density decreases with mass!
- in particular, this is true for central density.

Dimensionless Stellar Structure Equations

- Using

$$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$$

  we obtain

$$L^{1-\frac{2(n-1)}{3(n+3)}} \propto T_{\mathrm{eff}}^4$$

- for $n = 4$ we obtain

$$\log L = 5.6 \log T_{\mathrm{eff}} + \mathrm{const.}$$

- for $n = 16$ we obtain

$$\log L = 8.4 \log T_{\mathrm{eff}} + \mathrm{const.}$$

Dimensionless Stellar Structure Equations

- from $T_* = \frac{\mu G}{\mathcal{R}} \frac{M}{R_*}$ and $R_* \propto M^{\frac{n-1}{n+3}}$ we obtain for the (central) temperature

$$T_\mathrm{c} \propto M^{\frac{4}{n+3}}$$

- for $n = 4$ (pp chain, low-mass stars) we hence have

$$T_\mathrm{c} \propto M^{4/7}$$

- for $n = 16$ (CNO cycle, massive stars) we hence have

$$T_\mathrm{c} \propto M^{1/5}$$

- $\Rightarrow$ due to high temperature-sensitivity of CNO cycle nuclear burning, massive stars require only little higher central temperature to compensate for their hight luminosity.

Dimensionless Stellar Structure Equations

- calibration to the sun, $T_{\mathrm{c},\odot} \approx 1.5\times10^{7}\,\mathrm{K}$

$$\frac{T_\mathrm{c}}{T_{\mathrm{c},\odot}} = \left(\frac{M}{\mathrm{M}_\odot}\right)^{4/7}$$

- assuming minimum temperature T_min for hydrogen ignition, $T_\mathrm{min} \approx 4\times10^{6}\,\mathrm{K}$, and requiring $T_\mathrm{c} > T_\mathrm{min}$ we obtain

$$\frac{M}{\mathrm{M}_\odot} \geq \left(\frac{T_\mathrm{min}}{T_{\mathrm{c},\odot}}\right)^{7/4}$$

- $\Rightarrow$ minimum stellar mass $M_\mathrm{min} \approx 0.1\,\mathrm{M}_\odot$
- $\Rightarrow$ minimum stellar luminosity:

$$\frac{L_\mathrm{min}}{\mathrm{L}_\odot} = \left(\frac{M_\mathrm{min}}{\mathrm{M}_\odot}\right)^{3} \approx 10^{-3}$$