Astrophysics I: Stars and Stellar Evolution

AST 4001

Alexander Heger[1,2,3]

[1]School of Physics and Astronomy
University of Minnesota

[2]Nuclear & Particle Physics, Astrophysics & Cosmology Group, T-2
Los Alamos National Laboratory

[3]Department of Astronomy and Astrophysics
University of California at Santa Cruz

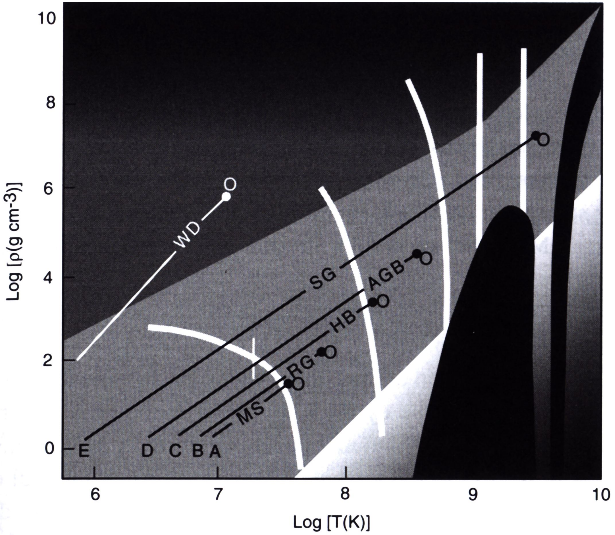
Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - Regimes of Stellar Evolution
   - Theory of the Main Sequence

2. Main Sequence Evolution
   - Concluding Remarks

Configuration of a 10 $M_\odot$ Star



Configuration of a 10 $M_\odot$ star at different evolution phases in the temperature-density diagram.

Stellar Structure Equations

stationary terms   time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \tag{2}$$

$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{3}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \tag{4}$$

$$\frac{\partial X_i}{\partial t} = f_i(\rho, T, \mathbf{X}) \tag{5}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Simplified Stellar Structure Equations

- only radiative temperature gradient: $\nabla = \nabla_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa FP}{mT^4}$
- only simple law for nuclear burning: $\varepsilon = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu = q_0\rho T^n$
- only ideal gas pressure: $P = P_{\mathrm{gas}} = \frac{\mathcal{R}T\rho}{\mu}$

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2\rho}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

$$\frac{\partial T}{\partial m} = -\frac{3}{4ac}\frac{\kappa}{T^3}\frac{F}{(4\pi r^2)^2}$$

$$\frac{\partial F}{\partial m} = q_0\rho T^n$$

$$P = \frac{\mathcal{R}T\rho}{\mu}$$

Dimensionless Stellar Structure Equations

- all functions $r(m)$, $P(m)$, $\rho(m)$, $T(m)$, and $F(m)$ need to be solved in range $0 \leq m \leq M$
- free parameter: mass M
- parameters κ, q_0, μ, and n determined from physics
- introduce dimension-less variable x with $0 \leq x \leq 1$:

$$x = \frac{m}{M}$$

- we can now write a set of dimension-less equations with functions $f_i(x)$ for these 5 quantities:

$$r = f_1(x) R_*$$
$$P = f_2(x) P_*$$
$$\rho = f_3(x) \rho_*$$
$$T = f_4(x) T_*$$
$$F = f_5(x) F_*$$

Dimensionless Stellar Structure Equations

- substituting
  $m = Mx$,
  $r = f_1(x) R_*$,
  $P = f_2(x) P_*$ into

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

  we obtain

$$\frac{P_*}{M}\frac{\mathrm{d}f_2}{\mathrm{d}x} = -\frac{GMx}{4\pi f_1^4 R_*^4}$$

- If we define

$$P_* = \frac{GM^2}{R_*^4}$$

  we may write

$$\frac{\mathrm{d}f_2}{\mathrm{d}x} = -\frac{x}{4\pi f_1^4}$$

Dimensionless Stellar Structure Equations

In a similar way we can re-write the entire set

$$\frac{\mathrm{d}f_2}{\mathrm{d}x} = -\frac{x}{4\pi f_1^4} \quad , \quad P_* = \frac{GM^2}{R_*^4}$$

$$\frac{\mathrm{d}f_1}{\mathrm{d}x} = \frac{1}{4\pi f_1^2 f_3} \quad , \quad \rho_* = \frac{M}{R_*^3}$$

$$f_2 = f_3 f_4 \quad , \quad T_* = \frac{\mu P_*}{\mathcal{R}\rho_*}$$

$$\frac{\mathrm{d}f_4}{\mathrm{d}x} = -\frac{3f_5}{4f_4^3\left(4\pi f_1^2\right)^2} \quad , \quad F_* = \frac{ac}{\kappa}\frac{T_*^4 R_*^4}{M}$$

$$\frac{\mathrm{d}f_5}{\mathrm{d}x} = f_3 f_4^n \quad , \quad F_* = q_0 \rho_* T_*^n M$$

$\Rightarrow$ *homology* of solution as function of M (and q_0, n, μ, and κ)

Dimensionless Stellar Structure Equations

- substituting $P_* = \frac{GM^2}{R_*^4}$ and $\rho_* = \frac{M}{R_*^3}$ into $T_* = \frac{\mu P_*}{\mathcal{R}\rho_*}$ we obtain:

$$T_* = \frac{\mu G}{\mathcal{R}} \frac{M}{R_*}$$

- adding this into $F_* = \frac{ac}{\kappa} \frac{T_*^4 R_*^4}{M}$ we obtain

$$F_* = \frac{ac}{\kappa} \left(\frac{\mu G}{\mathcal{R}} \right)^4 M^3$$

- we recover $L \propto M^3$
- $\tau_{\mathrm{MS}} = \frac{M}{L} \propto M^{-2}$
- but this relation also holds for any value of x - inside star at same relative mass coordinate

Dimensionless Stellar Structure Equations

- substituting $F_* = \frac{ac}{\kappa}\left(\frac{\mu G}{\mathcal{R}}\right)^4 M^3$ into $F_* = q_0 \rho_* T_*^n M$ and using $P_* = \frac{GM^2}{R_*^4}$, $\rho_* = \frac{M}{R_*^3}$, and $T_* = \frac{\mu P_*}{\mathcal{R}\rho_*}$ we obtain:

$$R_* \propto M^{\frac{n-1}{n+3}}$$

- $\Rightarrow$ for large n (CNO cycle: $n \approx 14 \ldots 16$):
roughly $R_* \propto M$
(big stars)
- $\Rightarrow$ for small n (pp chains: $n = 4$):
$R \propto M^{3/7}$

Dimensionless Stellar Structure Equations

- inserting $R_* \propto M^{\frac{n-1}{n+3}}$ into $\rho_* = \frac{M}{R_*^3}$ we obtain

$$\rho_* \propto M^{2\frac{3-n}{3+n}}$$

- since $n > 3$:
  density decreases with mass!
- in particular, this is true for central density.

Dimensionless Stellar Structure Equations

- Using

$$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$$

we obtain

$$L^{1-\frac{2(n-1)}{3(n+3)}} \propto T_{\mathrm{eff}}^4$$

- for $n = 4$ we obtain

$$\log L = 5.6 \log T_{\mathrm{eff}} + \mathrm{const.}$$

- for $n = 16$ we obtain

$$\log L = 8.4 \log T_{\mathrm{eff}} + \mathrm{const.}$$

Dimensionless Stellar Structure Equations

- from $T_* = \frac{\mu G}{\mathcal{R}} \frac{M}{R_*}$ and $R_* \propto M^{\frac{n-1}{n+3}}$ we obtain for the (central) temperature

$$T_{\mathrm{c}} \propto M^{\frac{4}{n+3}}$$

- for $n = 4$ (pp chain, low-mass stars) we hence have

$$T_{\mathrm{c}} \propto M^{4/7}$$

- for $n = 16$ (CNO cycle, massive stars) we hence have

$$T_{\mathrm{c}} \propto M^{1/5}$$

- $\Rightarrow$ due to high temperature-sensitivity of CNO cycle nuclear burning, massive stars require only little higher central temperature to compensate for their hight luminosity.

Dimensionless Stellar Structure Equations

- calibration to the sun, $T_{c,\odot} \approx 1.5\times10^7\,\mathrm{K}$

$$\frac{T_c}{T_{c,\odot}} = \left(\frac{M}{\mathrm{M}_\odot}\right)^{4/7}$$

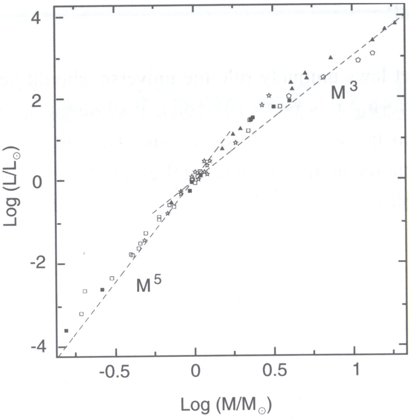
- assuming minimum temperature T_{min} for hydrogen ignition, $T_{min} \approx 4\times10^6\,\mathrm{K}$, and requiring $T_c > T_{min}$ we obtain

$$\frac{M}{\mathrm{M}_\odot} \geq \left(\frac{T_{min}}{T_{c,\odot}}\right)^{7/4}$$

- $\Rightarrow$ minimum stellar mass $M_{min} \approx 0.1\,\mathrm{M}_\odot$
- $\Rightarrow$ minimum stellar luminosity:

$$\frac{L_{min}}{\mathrm{L}_\odot} = \left(\frac{M_{min}}{\mathrm{M}_\odot}\right)^3 \approx 10^{-3}$$

Mass-Luminosity Relation for ZAMS Stars



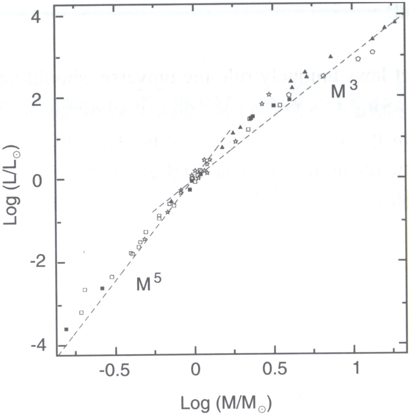
- We derived scaling law for the Main Sequence (MS) that scales as

$$L \propto M^3$$

- What causes deviation for low and high masses?
- Discuss with you neigbor and write down your solution.
- Please write your names on sheet and hand in (no grades).

Overview

Mass-Luminosity Relation for ZAMS Stars

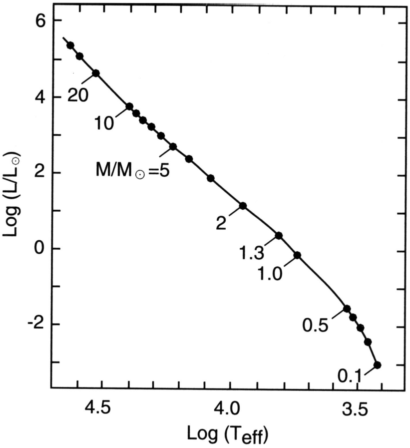


$$L \propto M^{\nu}$$

Notes:

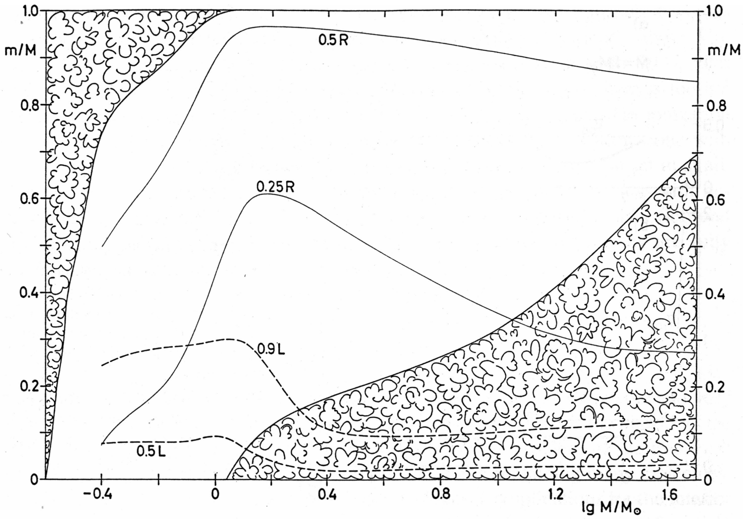
- $\nu \sim 3$ for stars dominated by ideal gas pressure
- for massive stars, $M \gtrsim 100\,\mathrm{M}_\odot$: $\nu \approx 1$ due to radiation pressure
- for low-mass stars: $\nu \approx 5$ due to (electron) gas degeneracy
- star is not purely radiative but also partly convective!

The Main-Sequence Phase



- when the star reaches the ZAMS the previous evolution is "forgotten"
- the structure of the star is uniquely defined by its mass and composition
- ...and rotation, magnetic fields

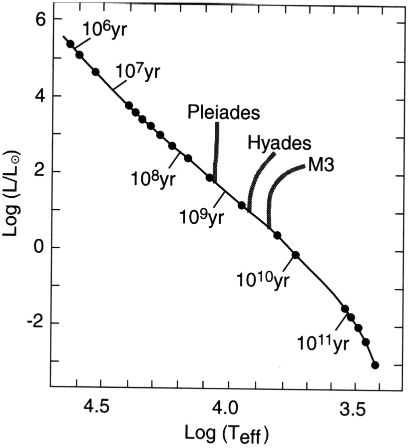
The Structure of Zero-Age Main-Sequence Stars



The Main-Sequence Evolution of Stars

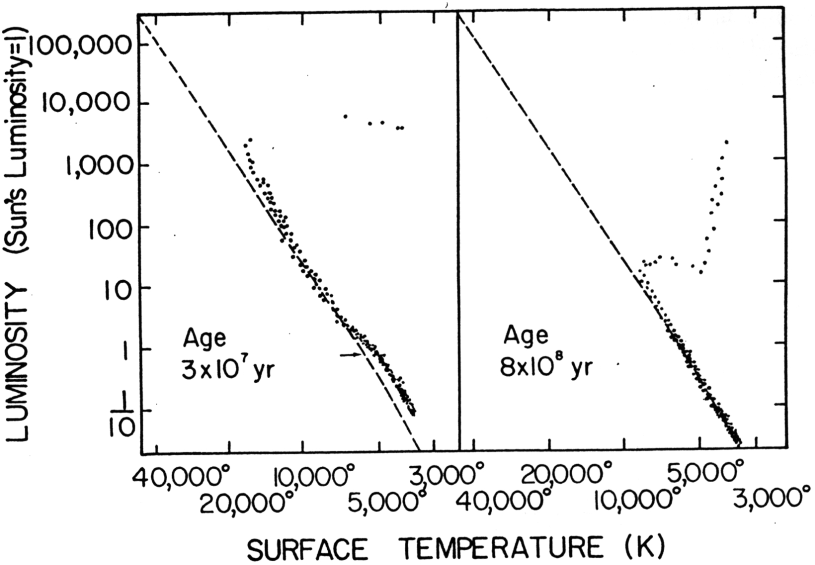
- nuclear burning proceeds in the interior of stars until hydrogen fuel is exhausted
- The center of the star contracts and heats up to compensate for reduction of fuel
- the outer layers of the star expand
- the lifetime is very strong function of the mass of the star
- convective regions evolve chemically homogeneously
- in some stars the products or results of nuclear burning in the can be found at the surface
  (Li depletion of the sun, N enrichment of massive stars)
- massive stars may lose significant amounts of mass due to stellar "winds"
- stars may also change their rotation rate - spin up or down.

The MS Phase in the HRD - Clusters

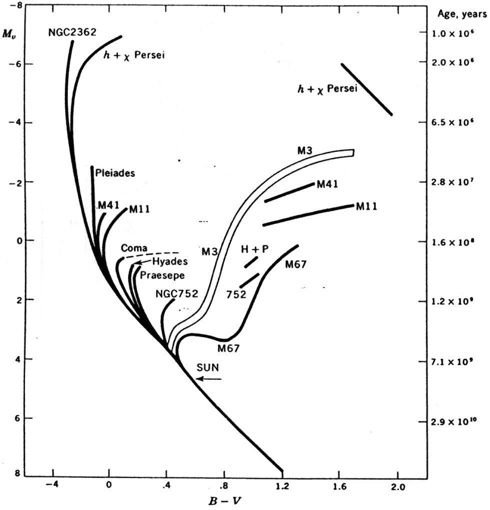


- stars evolve "off" the ZAMS as their evolution proceeds
- generally evolved stars are found to the right of the ZAMS
- isochrones define stars of equal age but different mass
- other factors can be composition (untypical), rotation, and binarity

The MS Phase in the HRD - Clusters



The MS Phase in the HRD - Clusters



- different clusters in the galaxy
- luminosity as a function of color
- turn-off determines cluster age

Main-Sequence lifetimes

Table 8.2 Main-sequence lifetimes

Mass ($M_\odot$)	*Time* (*yr*)	α
0.1	6×10^{12}	−2.8
0.5	7×10^{10}	−2.8
1.0	1×10^{10}	
1.25	4×10^{9}	−4.1
1.5	2×10^{9}	−4.0
3.0	2×10^{8}	−3.6
5.0	7×10^{7}	−3.1
9.0	2×10^{7}	−2.8
15	1×10^{7}	−2.6
25	6×10^{6}	−2.3

- logarithmic change of lifetime with (initial) mass

$$\alpha = \frac{\log\left(\tau_{\mathrm{MS}}/\tau_{\mathrm{MS},\odot}\right)}{\log\left(M/\mathrm{M}_\odot\right)}$$

- note large range in stellar lifetimes!
- maybe better use slope of lifetimes, $\tau_{\mathrm{MS}} \propto M^{\alpha'}$:

$$\alpha' = \frac{\mathrm{d}\,\ln \tau_{\mathrm{MS}}}{\mathrm{d}\,\ln M}$$

Main-Sequence lifetimes

NOTE

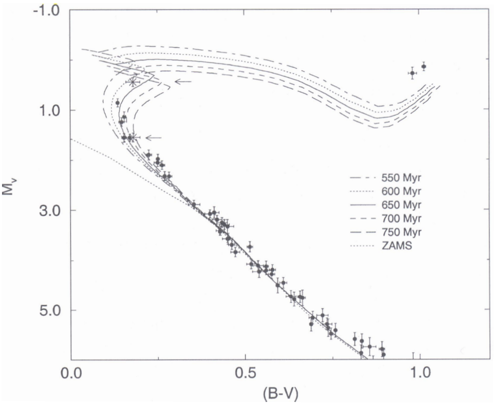
- stars with masses below $0.7\,M_\odot$ have not yet evolved off the MS even if as old as the universe!
  These are red dwarf stars. All ever formed are still around.
- stars with initial masses $M \lesssim 2\,M_\odot$ ignite helium burning under degenerate conditions in their core. They are usually referred to as low-mass stars.
- stars with initial mass $2M_\odot \lesssim 9M_\odot$ are called intermediate mass stars. They ignite helium burning non-degenerate.
  We can distinguish stars that later ignite carbon burning in the center ($M \gtrsim 7.5\,M_\odot$) and those that don't.
- Stars with masses $M \gtrsim 9\,M_\odot$ form iron codes that collapse to make core collapse supernovae

Main-Sequence lifetimes

NOTE (continued)

- very massive stars with $M \gtrsim 100\,\mathrm{M}_\odot$ may even die in different ways, as pair instability supernovae
- super-massive stars (if ever formed), $M \gtrsim 100,000\,\mathrm{M}_\odot$ may never reach the main sequence and collapse or explode due to general relativistic effects (adiabatic index needed for stability becomes greater then $4/3$)
- the initial-mass limits for low-mass stars, intermediatre-mass stars, and supernovae depend on composition of the stars. For example, for star of 10^{-4} solar metallicity, the lower mass limit for supernovae is lower by $\sim 1.5\,\mathrm{M}_\odot$
- There are significant uncertainties in the quoted mass limits depending on stellar models due to our limited understanding of mixing processes inside stars, up to $\sim 1\,\mathrm{M}_\odot$: the lower mass limit for supernovae is somewhere between $8\,\mathrm{M}_\odot$ and $10\,\mathrm{M}_\odot$ at solar metallicity

Main Sequance in a Star Cluster



Hyades cluster and stellar tracks

$$\log L = \alpha \log T_{\mathrm{eff}} + \mathrm{const.}$$