Astrophysics I: Stars and Stellar Evolution

AST 4001

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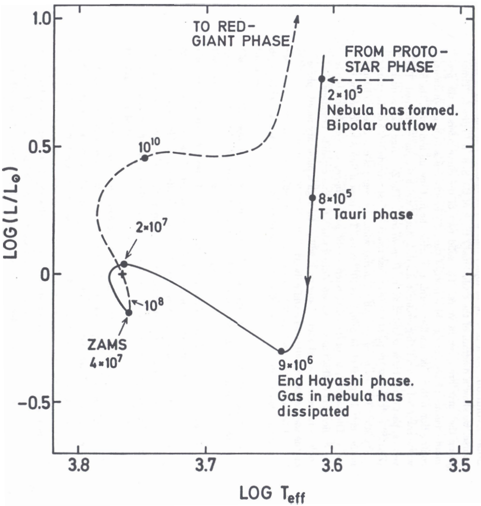
Stars and Stellar Evolution, Fall 2008

Overview

Evolution of the Sun in the HRD



Evolution of the sun from formation through hydrogen burning

Stellar Populations

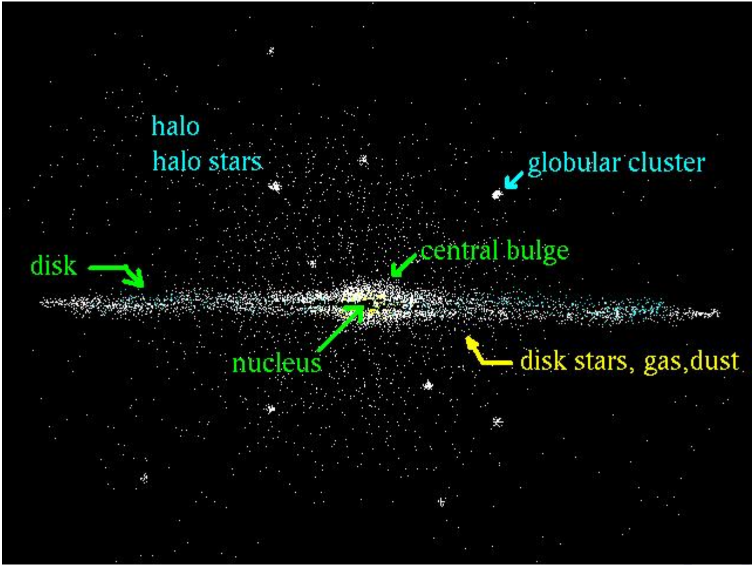
We distinguish stars by location (in our galaxy)

- halo stars
- thick disk stars
- thin disk stars
- bulge stars

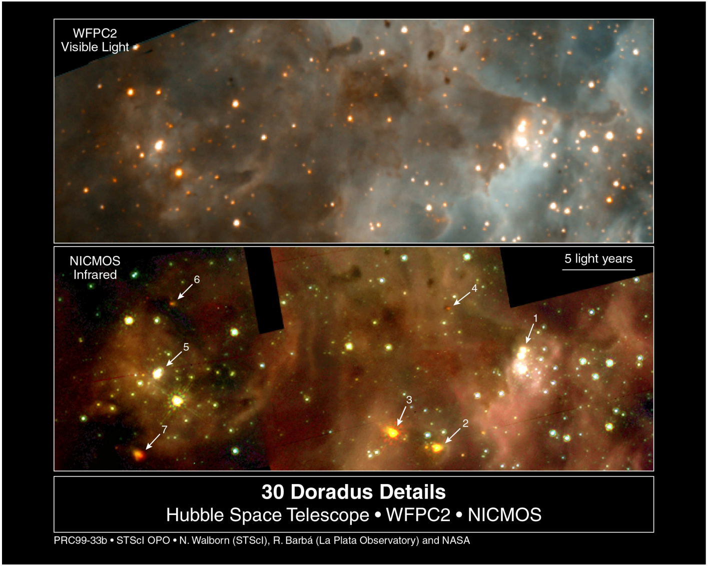
and by formation time/composition

- Population I: modern stars the form today, like the sun
- Population II: old stars, in halo, low metalicity
- Population III: the first generation of primordial stars

Structure of the Milky Way



(Zoom into 30 Dorados)



Zoom into 30 Dor

The Initial Mass Function

- Observationally, the mass spectrum of stars formed - the relative number of star made at a given mass - seems to be universal (except likely the first generation of stars where we do not have observational data yet)
- Independent of galactic age or location
- The number of stars in a mass bin $[M, M + \mathrm{d}M]$ can be written defining the birth function $\Phi(M)$:

$$\mathrm{d}N = \Phi(M)\mathrm{d}M$$

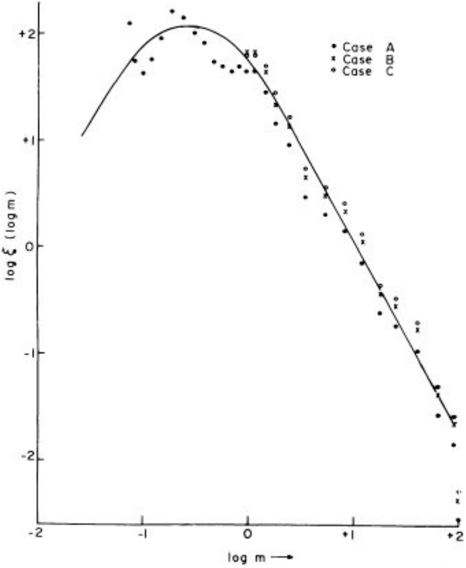
- The mass of stars in a mass bin is then given by weighing by mass M, defining the initial mass function (**IMF**) $\xi(M)$:

$$\xi(M) = M\mathrm{d}N/\mathrm{d}M$$

- Salpeter (1955) found observationally a power law for Φ, ξ:

$$\Phi(M) \propto M^{-2.35}\ , \quad \xi(M) \propto M^{-1.35}$$

IMF



IMF Notes

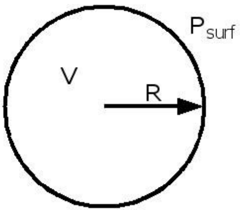
NOTE

- the distribution of stellar masses we see today (or at any given time) is not that of the birth function because stars have a finite lifetime.
- if written as

  $$\xi(M) \propto M^{-\gamma}$$

  even today best data indicates an almost global law of $\gamma = 1.5 \pm 0.3$.
- at low mass, below around $0.3\,M_\odot$, we find a break in the IMF and there seem to be fewer objects of low mass - the IMF appears to get "flat" ($\gamma = 0$).
- the total mass of stars below $0.3\,M_\odot$ seems to be less than 20 % of that above

Star Formation - Jeans Mass



- star formation generally in turbulent medium
- assume hydrostatic equilibrium
- reaction of a region to a perturbation will depend on dynamic stability of that region
- assume spherical region of volume V and surface pressure P_{surf}
- partial virial theorem

$$P_{\mathrm{s}} V_{\mathrm{s}} - \int_0^{M_{\mathrm{s}}} \frac{P}{\rho}\,\mathrm{d}m = \frac{1}{3}\Omega_{\mathrm{s}} \quad \text{becomes}$$

$$\int P \mathrm{d}V = P_{\mathrm{surf}} V + \frac{1}{3}\alpha\frac{GM^2}{R}$$

Jeans Radius

- assuming ideal gas we obtain

$$\int P \mathrm{d} V = \frac{\mathcal{R}}{\mu} T \int \rho \mathrm{d} V = \frac{\mathcal{R}}{\mu} T M$$

  and hence

$$\frac{\mathcal{R}}{\mu} T M = P_{\mathrm{surf}} V + \frac{1}{3} \alpha \frac{G M^2}{R}$$

- since $P_{\mathrm{surf}} V > 0$ we can obtain an minimum radius for stability

$$R \geq \frac{\alpha}{3} \frac{\mu G M}{\mathcal{R} T}$$

  and define accordingly the Jeans radius

$$R_{\mathrm{Jeans}} := \frac{\alpha}{3} \frac{\mu G M}{\mathcal{R} T}$$

  after Sir James H. Jeans.

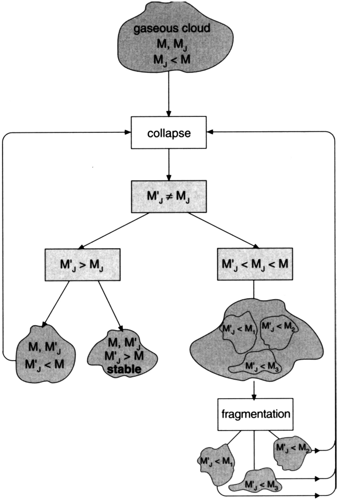
Jeans Mass

- A cloud with smaller radius, i.e., higher density will be unstable!
- Assuming an average density $\rho = M/V$, $V = \frac{4\pi}{3}R^3$, we can derive a critical mass as a function of temperature and density of the cloud, the Jeans Mass:

$$M_{\text{Jeans}} = \left[\left(\frac{3}{4\pi}\right)^{1/2}\left(\frac{3}{\alpha}\right)^{3/2}\right]\left(\frac{\mathcal{R}T}{\mu G}\right)^{3/2}\rho^{-1/2} \approx 10^5\,\mathrm{M}_\odot\sqrt{\frac{T^3}{n}}$$

  where n is the number density of gas particles.
- For typical galactic values we obtain Jeans mass of the order of thousands of stellar masses.

Collapse and Fragmentation



When collapse is started, the further evolution is determined by cooling.

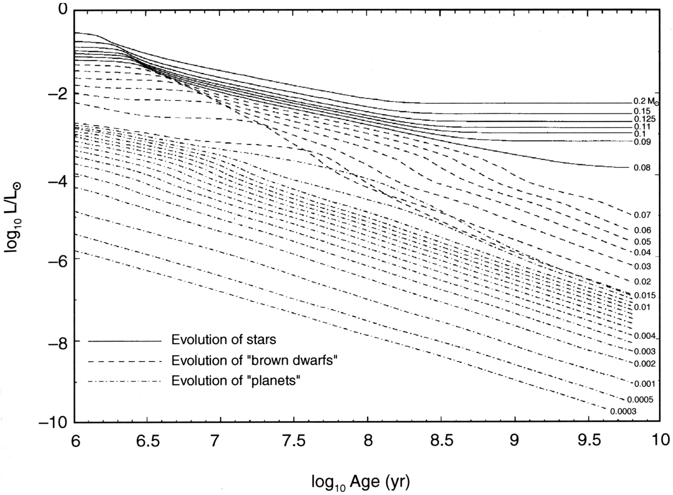
- if cooling is inefficient Jeans mass needs to increase (by accretion) to allow further collapse
- if cooling is efficient, the Jeans mass decreases and the collapsing cloud could fragment, allowing to make smaller stars.

Complications:

- magnetic fields, turbulence, shocks, heating by irradiation
- angular momentum and its transport (rotation, centrifugal force)

Eagle Nebula - current region of star formation

Luminosity Evolution as a Function of Mass



A Fully Convective Star

- consider fully convective star
- temperature gradient is very close to adiabatic temperature gradient hence we can describe the star by a polytrope with adiabatic index n given by:

$$n = \left(\frac{1}{\gamma_{\mathrm{ad}} - 1}\right) , \quad P = K\rho^{\gamma_{\mathrm{ad}}} = K\rho^{1+\frac{1}{n}}$$

- The constant K is given by (Lane-Emden M-R relation)

$$K^n = C_n G^n M^{n-1} R^{3-n} , \quad C_n = \frac{4\pi}{(n+1)^n} \frac{R_n^{n-3}}{M_n^{n-1}}$$

where again R_n, M_n come from the (tabulated) solution of the Lane-Emden equation.

Hayashi Zone

- One free parameter: outer boundary, radius of star R, "photosphere"
- assume hydrostatic equilibrium and integrating outward:

$$\frac{\mathrm{d}P}{\mathrm{d}r} \approx -\rho\frac{GM}{R^2}\,, \quad P_R = \frac{GM}{R^2}\int_R^\infty \rho\,\mathrm{d}r$$

- Assume temperature is given by the luminosity of the star, $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$, the optical depth of the surface is of order unity, and opacity is given by $\kappa = \kappa_0 \rho^a T^b$, we have

$$1 \approx \int_R^\infty \kappa\rho \mathrm{d}r = \bar{\kappa}\int_R^\infty \rho \mathrm{d}r \approx \kappa_0 \rho(R)^a T_{\mathrm{eff}}^b \int_R^\infty \rho \mathrm{d}r$$

- eliminating the integral we obtain

$$P_R = \frac{GM}{R^2 \kappa_0}\frac{1}{\rho(R)^a T_{\mathrm{eff}}^b}$$

Hayashi Zone

Combined with the EOS for the ideal gas, $P_R = \mathcal{R}\rho T/\mu$ we have a set of of four equations

$$
\begin{aligned}
\log P_R &= \log M - 2\log R - a\log\rho(R) - b\log T_{\text{eff}} + \text{const.} \\
n\log P_R &= (n-1)\log M + (3-n)\log R + (n+1)\log\rho(R) + \text{const.} \\
\log P_R &= \log\rho(R) + \log T_{\text{eff}} + \text{const.} \\
\log L &= 2\log R + 4\log T_{\text{eff}} + \text{const.}
\end{aligned}
$$

This gives

$$\log L = A\log T_{\text{eff}} + B\log M + \text{const.}\ ,$$

$$A = \frac{(7-n)(a+1) - 4 - a + b}{0.5(3-n)(a+1) - 1}\ , \quad B = \frac{(n-1)(a+1) + 1}{0.5(3-n)(a+1) - 1}$$

or, for $a = 1$ (reasonable assumption)

$$A = \frac{9 - 2n + b}{2 - n}\ , \quad B = \frac{2n - 1}{2 - n}$$

Interpretation of the Hayashi Zone

- dynamic stability: $n < 3$
  therefore $1.5 \leq n < 3$
- for $b = 4$, $n = 1.5$ (ideal mono-atomic gas)
  we have $A = 20$!
  $\Rightarrow$ almost vertical lines
  $\Rightarrow$ tracks for different stellar masses lie very closely together
  $\Rightarrow$ this region of the HRD is called Hayashi zone/line
- for fully convective star $\bar{\gamma} = \bar{\gamma_{\mathrm{ad}}}$
  $\bar{\gamma} > \bar{\gamma_{\mathrm{ad}}}$ would require super-adiabatic star
  "forbidden" regime right of the Hayashi line