Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - Stellar Populations and Initial Mass Function
   - Pre-MS Evolution
2. Pre-MS Evolution
   - Pre-MS Lifetimes
3. Low-Mass and Intermediate-Mass Stars
   - Core Helium Ignition

Stellar Populations

We distinguish stars by location (in our galaxy)

- halo stars
- thick disk stars
- thin disk stars
- bulge stars

and by formation time/composition

- Population I: modern stars the form today, like the sun
- Population II: old stars, in halo, low metalicity
- Population III: the first generation of primordial stars

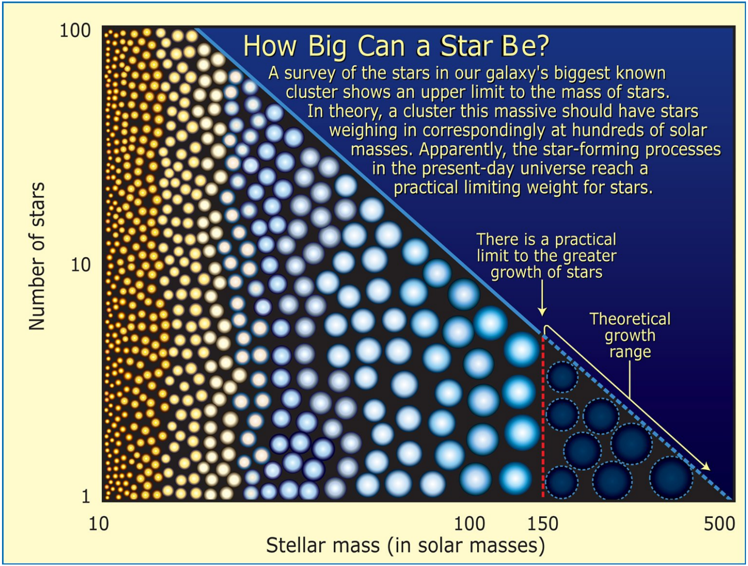
Summary: IMF, Jeans Mass

- birth function and initial mass function (Salpeter 1955)

$$\Phi(M) \propto M^{-2.35}\,, \quad \xi(M) \propto M^{-1.35}$$

- Jeans Mass:

$$M_{\mathrm{Jeans}} = \left[\left(\frac{3}{4\pi}\right)^{1/2}\left(\frac{3}{\alpha}\right)^{3/2}\right]\left(\frac{\mathcal{R}T}{\mu G}\right)^{3/2}\rho^{-1/2} \approx 10^5\,\mathrm{M}_\odot\sqrt{\frac{T^3}{n}}$$


How Big Can a Star Be?
A survey of the stars in our galaxy's biggest known cluster shows an upper limit to the mass of stars. In theory, a cluster this massive should have stars weighing in correspondingly at hundreds of solar masses. Apparently, the star-forming processes in the present-day universe reach a practical limiting weight for stars.
There is a practical limit to the greater growth of stars
Theoretical growth range
Number of stars
100
10
1
10
100
150
500
Stellar mass (in solar masses)

A Fully Convective Star

- consider fully convective star
- temperature gradient is very close to adiabatic temperature gradient hence we can describe the star by a polytrope with adiabatic index n given by:

$$n = \left(\frac{1}{\gamma_{\mathrm{ad}} - 1}\right)\,, \qquad P = K\rho^{\gamma_{\mathrm{ad}}} = K\rho^{1+\frac{1}{n}}$$

- The constant K is given by (Lane-Emden M-R relation)

$$K^n = C_n G^n M^{n-1} R^{3-n}\,, \qquad C_n = \frac{4\pi}{(n+1)^n}\frac{R_n^{n-3}}{M_n^{n-1}}$$

where again R_n, M_n come from the (tabulated) solution of the Lane-Emden equation.

Hayashi Zone

- One free parameter: outer boundary, radius of star R, "photosphere"
- assume hydrostatic equilibrium and integrating outward:

$$\frac{\mathrm{d}P}{\mathrm{d}r} \approx -\rho \frac{GM}{R^2}\,, \qquad P_R = \frac{GM}{R^2} \int_R^\infty \rho\,\mathrm{d}r$$

- Assume temperature is given by the luminosity of the star, $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$, the optical depth of the surface is of order unity, and opacity is given by $\kappa = \kappa_0 \rho^a T^b$, we have

$$1 \approx \int_R^\infty \kappa \rho \mathrm{d}r = \bar{\kappa} \int_R^\infty \rho \mathrm{d}r \approx \kappa_0 \rho(R)^a T_{\mathrm{eff}}^b \int_R^\infty \rho \mathrm{d}r$$

- eliminating the integral we obtain

$$P_R = \frac{GM}{R^2 \kappa_0} \frac{1}{\rho(R)^a T_{\mathrm{eff}}^b}$$

Hayashi Zone

Combined with the EOS for the ideal gas, $P_R = \mathcal{R}\rho T/\mu$ we have a set of of four equations

$$\begin{aligned}
\log P_R &= \log M - 2\log R - a\log\rho(R) - b\log T_\mathrm{eff} + \mathrm{const.} \\
n\log P_R &= (n-1)\log M + (3-n)\log R + (n+1)\log\rho(R) + \mathrm{const.} \\
\log P_R &= \log\rho(R) + \log T_\mathrm{eff} + \mathrm{const.} \\
\log L &= 2\log R + 4\log T_\mathrm{eff} + \mathrm{const.}
\end{aligned}$$

This gives

$$\log L = A\log T_\mathrm{eff} + B\log M + \mathrm{const.}\ ,$$

$$A = \frac{(7-n)(a+1) - 4 - a + b}{0.5(3-n)(a+1) - 1}\ , \quad B = \frac{(n-1)(a+1)+1}{0.5(3-n)(a+1)-1}$$

or, for $a = 1$ (reasonable assumption)

$$A = \frac{9-2n+b}{2-n}\ , \quad B = \frac{2n-1}{2-n}$$

Interpretation of the Hayashi Zone

- dynamic stability: $n < 3$
  therefore $1.5 \leq n < 3$
- for $b = 4$, $n = 1.5$ (ideal mono-atomic gas)
  we have $A = 20$!
  $\Rightarrow$ almost vertical lines
  $\Rightarrow$ tracks for different stellar masses lie very closely together
  $\Rightarrow$ this region of the HRD is called Hayashi zone/line
- for fully convective star $\bar{\gamma} = \bar{\gamma_{\rm ad}}$
  $\bar{\gamma} > \bar{\gamma_{\rm ad}}$ would require super-adiabatic star
  "forbidden" regime right of the Hayashi line

Overview

pre-Main Sequence Evolution

- assume cloud contracts on free fall time scale at first

$$\tau_{\rm ff} \sim \frac{1}{\sqrt{G\rho}}$$

  and releases gravitational energy of $\alpha GM^2/R$ during contraction.

- at first, this is used up to dissociate H_2 and ionize the hydrogen and helium.
- assume dissociation and ionization potentials $\chi_{\rm H_2} = 4.5\,\mathrm{eV}$, $\chi_{\rm H} = 13.6\,\mathrm{eV}$, and $\chi_{\rm He} = 79\,\mathrm{eV}$, then we have

$$\alpha\frac{GM^2}{R} \approx \frac{M}{\rm u}\left(\frac{X}{2}\chi_{\rm H_2} + X\chi_{\rm H} + \frac{Y}{4}\chi_{\rm He}\right)$$

pre-Main Sequence Evolution

- Using $Y \approx 1 - X$ ($Z \sim 0.02$), $\alpha \approx 0.5$ we have

$$\frac{R}{\mathrm{R}_\odot} \approx \frac{50}{1 - 0.2X} \frac{M}{\mathrm{M}_\odot}$$

- using the virial theorem and $X \approx 0.7$ we get an average temperature of the star of

$$\bar{T} = \frac{\alpha\mu}{3k_\mathrm{B}} \frac{GMm_\mathrm{H}}{R} \approx 6\times10^4\,\mathrm{K}$$

pre-Main Sequence Evolution

- After starting off on the Hayashi line convection recedes inside the star
- the continues to contract and heat up
- eventually nuclear burning is ignited
- this halts further contraction
- the star reaches hydrostatic *and* thermal equilibrium
- this defines the Zero-Age Main Sequence (ZAMS)

pre-Main Sequence Evolution

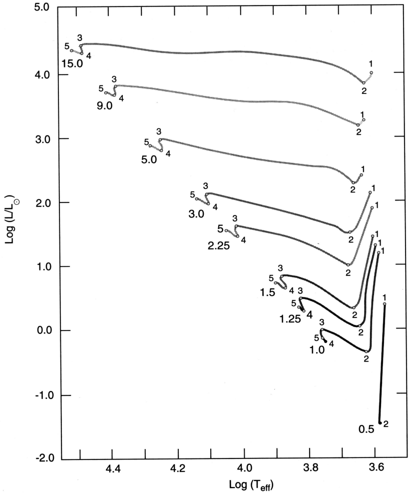
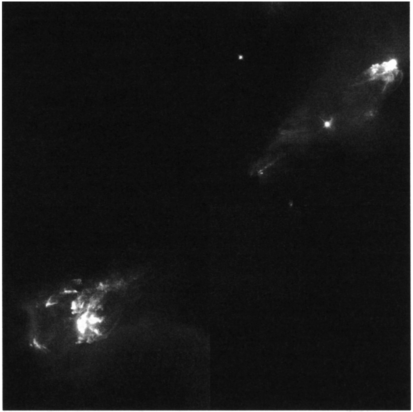


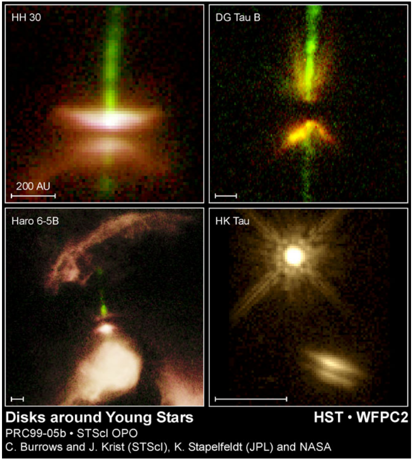
Table 8.1 Evolutionary lifetimes (years)

$M/M_\odot$	*1–2*	*2–3*	*3–4*	*4–5*
15	6.7(2)	2.6(4)	1.3(4)	6.0(3)
9	1.4(3)	7.8(4)	2.3(4)	1.8(4)
5	2.9(4)	2.8(5)	7.4(4)	6.8(4)
3	2.1(5)	1.0(6)	2.2(5)	2.8(5)
2.25	5.9(5)	2.2(6)	5.0(5)	6.7(5)
1.5	2.4(6)	6.3(6)	1.8(6)	3.0(6)
1.25	4.0(6)	1.0(7)	3.5(6)	1.0(7)
1.0	8.9(6)	1.6(7)	8.9(6)	1.6(7)
0.5	1.6(8)			

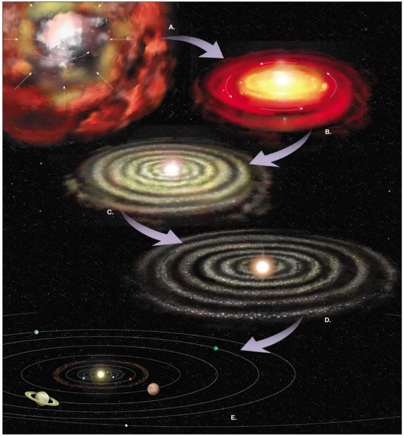
Note: powers of 10 are given in parentheses.

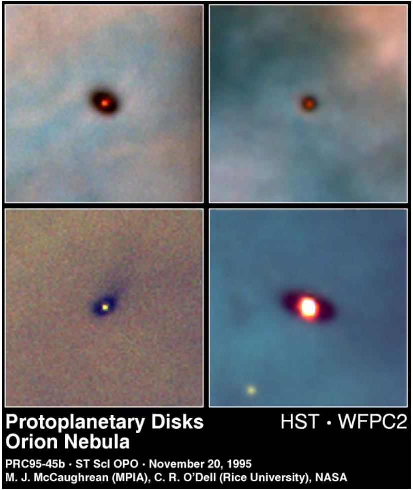
T Tauri outflows





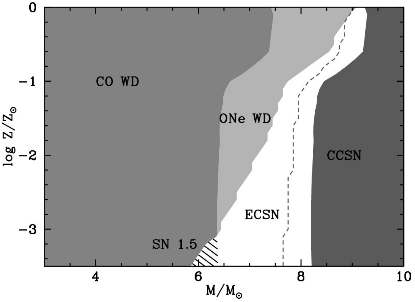
Star formation and disks





Overview

Fates of Intermediate and Massive Stars



NOTE

- exact mass ranges depend on uncertainties in stellar physics (mixing, mass loss, nuclear cross sections, etc.)
- they also depend on metallicity and on initial rotation rate of star

The Red Giant Phase

1. After end of central hydrogen burning massive stars contract and ignite helium and later burning stages till they build up an iron core. That core eventually collapse sand a supernova results while a neutron star or black hole is left behind.
2. Stars of lower mass eventually become degenerate and contraction is halted. They end their evolution before the nuclei are fused all the way to iron. The result are white dwarf stars - made of Ne+O+Mg or C+O.
3. In the lightest stars even helium becomes degenerate before helium burning is ignited.

The Schönberg-Chandrasekhar Limit

- Low mass stars have a radiative core.
- hydrogen first depletes in the center, then increasingly further out
- this leads to the gradual build-up of a non-degenerate helium core of increasing mass.
- a critical limit exists above which this core no longer can sustain the pressure against the overlaying envelope layers, the The Schönberg-Chandrasekhar Limit.

Derivation of the Schönberg-Chandrasekhar Limit (1)

- Assume a core of mass M_c, radius R_c, volume V_c, mean molecular weight μ_c, temperature T_c, surface pressure P_s at its outer boundary
- The *partial* virial theorem then is

$$\int_0^{V_\mathrm{c}} P\,\mathrm{d}V = P_\mathrm{s} V_\mathrm{c} + \frac{\alpha}{3}\frac{G{M_\mathrm{c}}^2}{R_\mathrm{c}}$$

- for an ideal gas this becomes

$$\int_0^{V_\mathrm{c}} P\,\mathrm{d}V = \int_0^{V_\mathrm{c}} \frac{\mathcal{R}T\rho}{\mu}\,\mathrm{d}V = \frac{\mathcal{R}T_\mathrm{c}}{\mu_\mathrm{c}} \int_0^{V_\mathrm{c}} \rho\,\mathrm{d}V = \frac{\mathcal{R}}{\mu_\mathrm{c}} T_\mathrm{c} M_\mathrm{c}$$

- Using $V_\mathrm{c} = \frac{4\pi}{3}{R_\mathrm{c}}^3$ we obtain

$$P_\mathrm{s}(R_\mathrm{c}) = \frac{3}{4\pi}\frac{\mathcal{R}T_\mathrm{c}}{\mu_\mathrm{c}}\frac{M_\mathrm{c}}{{R_\mathrm{c}}^3} - \frac{\alpha G}{4\pi}\frac{{M_\mathrm{c}}^2}{{R_\mathrm{c}}^4}$$

Derivation of the Schönberg-Chandrasekhar Limit (II)

- in the limit of $P_\mathrm{s} = 0$ we have a radius of

$$R_0 = \frac{\alpha}{3} \frac{G}{\mathcal{R}} \frac{M_\mathrm{c} \mu_\mathrm{c}}{T_\mathrm{c}}$$

- a maximum pressure can be derived from $\mathrm{d}P_\mathrm{s}/\mathrm{d}R_\mathrm{c} = 0$

$$R_1 = \frac{4\alpha G}{9\mathcal{R}} \frac{M_\mathrm{c} \mu_\mathrm{c}}{T_\mathrm{c}}$$

- for a core radius $R_\mathrm{c} < R_0$ the core would collapse under its own gravity $\Rightarrow$ helium ignition
- for a core radius $R_\mathrm{c} > R_1$, P_s is smaller than its maximum, $P_\mathrm{s,max}$

$$P_\mathrm{s,max} = C_1 \frac{T_\mathrm{c}^4}{{M_\mathrm{c}}^2 {\mu_\mathrm{c}}^2} , \quad C_1 = \frac{\mathcal{R}^4}{\pi \alpha^3 G^3} \frac{3^7}{4^5}$$