Astrophysics I: Stars and Stellar Evolution

AST 4001

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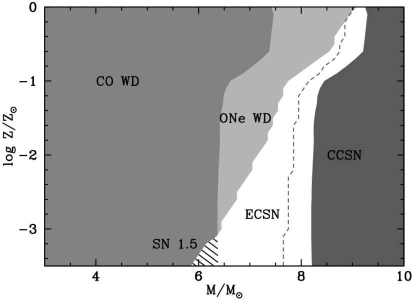
Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - Core Helium Ignition
2. Low-Mass and Intermediate-Mass Stars
   - Core Helium Ignition
   - The Herzsprung Gap
   - Evolution of a $8\,M_\odot$ Star

Fates of Intermediate and Massive Stars



NOTE

- exact mass ranges depend on uncertainties in stellar physics (mixing, mass loss, nuclear cross sections, etc.)
- they also depend on metallicity and on initial rotation rate of star

The Red Giant Phase

1. After end of central hydrogen burning massive stars contract and ignite helium and later burning stages till they build up an iron core. That core eventually collapse sand a supernova results while a neutron star or black hole is left behind.
2. Stars of lower mass eventually become degenerate and contraction is halted. They end their evolution before the nuclei are fused all the way to iron. The result are white dwarf stars - made of Ne+O+M or C+O.
3. In the lightest stars even helium becomes degenerate before helium burning is ignited.

The Schönberg-Chandrasekhar Limit

- Low mass stars have a radiative core.
- hydrogen first depletes in the center, then increasingly further out
- this leads to the gradual build-up of a non-degenerate helium core of increasing mass.
- a critical limit exists above which this core no longer can sustain the pressure against the overlaying envelope layers, the The Schönberg-Chandrasekhar Limit.

Derivation of the Schönberg-Chandrasekhar Limit (1)

- Assume a core of mass M_c, radius R_c, volume V_c, mean molecular weight μ_c, temperature T_c, surface pressure P_s at its outer boundary
- The *partial* virial theorem then is

$$\int_0^{V_\mathrm{c}} P\,\mathrm{d}V = P_\mathrm{s} V_\mathrm{c} + \frac{\alpha}{3}\frac{G {M_\mathrm{c}}^2}{R_\mathrm{c}}$$

- for an ideal gas this becomes

$$\int_0^{V_\mathrm{c}} P\,\mathrm{d}V = \int_0^{V_\mathrm{c}} \frac{\mathcal{R}T\rho}{\mu}\,\mathrm{d}V = \frac{\mathcal{R}T_\mathrm{c}}{\mu_\mathrm{c}}\int_0^{V_\mathrm{c}} \rho\,\mathrm{d}V = \frac{\mathcal{R}}{\mu_\mathrm{c}} T_\mathrm{c} M_\mathrm{c}$$

- Using $V_\mathrm{c} = \frac{4\pi}{3}{R_\mathrm{c}}^3$ we obtain

$$P_\mathrm{s}(R_\mathrm{c}) = \frac{3}{4\pi}\frac{\mathcal{R}T_\mathrm{c}}{\mu_\mathrm{c}}\frac{M_\mathrm{c}}{{R_\mathrm{c}}^3} - \frac{\alpha G}{4\pi}\frac{{M_\mathrm{c}}^2}{{R_\mathrm{c}}^4}$$

Derivation of the Schönberg-Chandrasekhar Limit (II)

- in the limit of $P_\mathrm{s}=0$ we have a radius of

$$R_0=\frac{\alpha}{3}\frac{G}{\mathcal{R}}\frac{M_\mathrm{c}\mu_\mathrm{c}}{T_\mathrm{c}}$$

- a maximum pressure can be derived from $\mathrm{d}P_\mathrm{s}/\mathrm{d}R_\mathrm{c}=0$

$$R_1=\frac{4\alpha G}{9\mathcal{R}}\frac{M_\mathrm{c}\mu_\mathrm{c}}{T_\mathrm{c}}$$

- for a core radius $R_\mathrm{c}<R_0$ the core would collapse under its own gravity $\Rightarrow$ helium ignition
- for a core radius $R_\mathrm{c}>R_1$, P_s is smaller than its maximum, $P_\mathrm{s,max}$

$$P_\mathrm{s,max}=C_1\frac{T_\mathrm{c}^4}{{M_\mathrm{c}}^2{\mu_\mathrm{c}}^2}\,,\qquad C_1=\frac{\mathcal{R}^4}{\pi\alpha^3G^3}\frac{3^7}{4^5}$$

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Derivation of the Schönberg-Chandrasekhar Limit (III)

- using the relation

$$P_\mathrm{c} > \frac{GM^2}{8\pi R^4}$$

- for a small core, $R_\mathrm{c} \ll R$ we approximate envelope pressure from

$$P_\mathrm{env} > \frac{GM^2}{8\pi R^4}$$

- $R_0 < R < R_1$ for $0 < P_\mathrm{s} < P_\mathrm{s,max}$
- to obtain an equilibrium configuration, $P_\mathrm{s,max}$ needs to be larger than the minimum pressure of the envelope

$$P_\mathrm{s,max} = C_1 \frac{T_\mathrm{c}^4}{{M_\mathrm{c}}^2 {\mu_\mathrm{c}}^2} \geq \frac{GM^2}{8\pi R^4}$$

Derivation of the Schönberg-Chandrasekhar Limit (IV)

- using the homolgy relation

$$T_c \propto \frac{\mu_{env} G}{\mathcal{R}} \frac{M}{R}$$

we can eliminate T_c and R and obtain

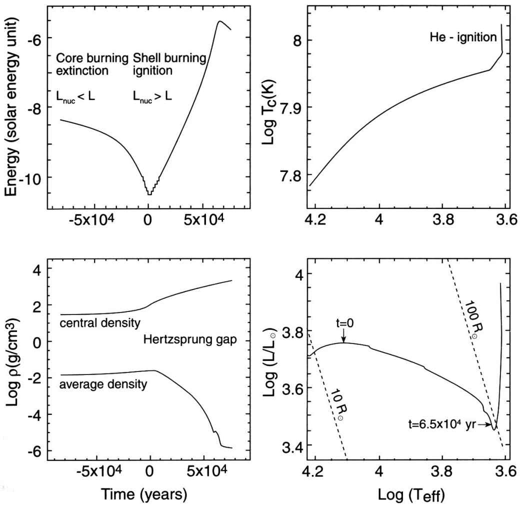
$$\frac{M_c}{M} \lesssim C_2 \left(\frac{\mu_{env}}{\mu_c}\right)^2, \quad C_2 \sim 0.37$$

- with $\mu_{env} \approx 0.75$, $\mu_c \approx 1.33$ one finds $M/M_c \lesssim 0.12$.
  $\Rightarrow$ beyond that limit the core starts contacting.

The Hertzsprung Gap

- Stars more massive than $\sim 2\,M_\odot$ have a convective core that is bigger than the Schönberg-Chandrasekhar limit. It contacts immediately after core hydrogen depletion until a stable temperature and density gradient is established.
- then shell hydrogen burning sets is
- high temperature sensitivity of CNO cycle allows energy generation larger than luminosity of the star
- the envelope of the star expands
- usually comparably short time scale
- fast transition in HRD leads to Hertzsprung Gap
- eventually cool envelope becomes convective
- formation of red giant star
- first dredge-up brings chemically processed material to the surface of the star

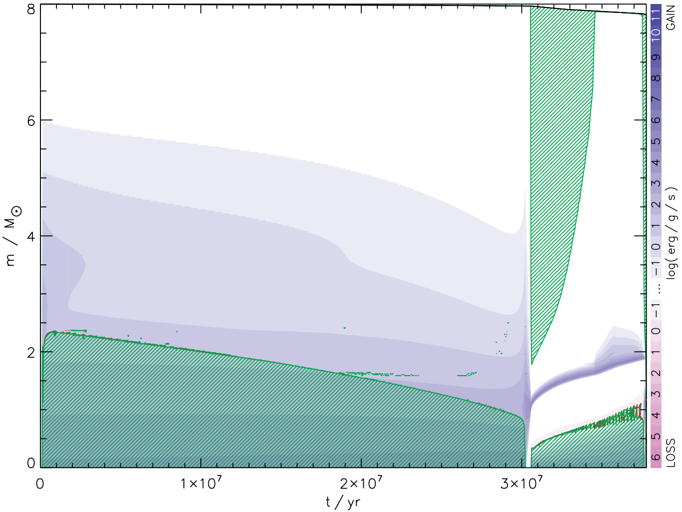
Evolution Through Core Contraction



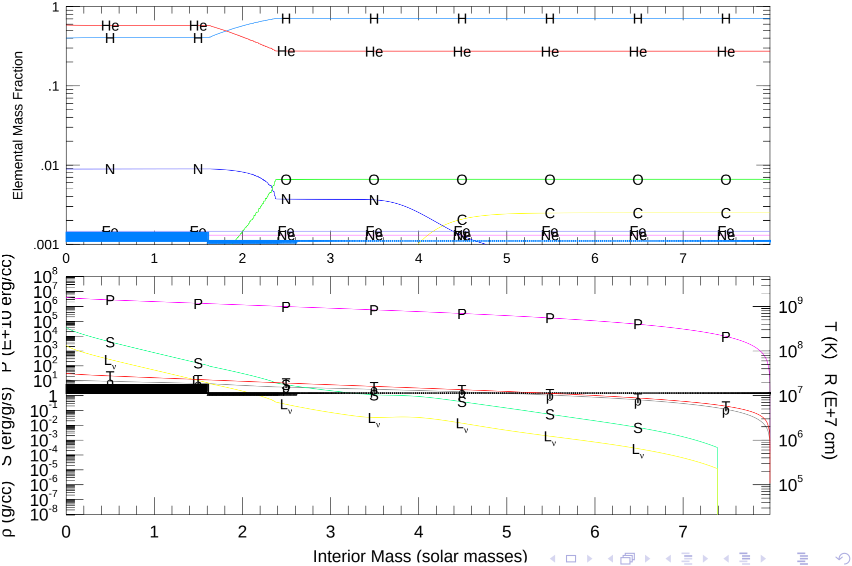
Note:

- Fast evolution too cool T_{eff}
- $\Rightarrow$ few stars are found in transition region
- $\Rightarrow$ "Hertzprung Gap" in HRD

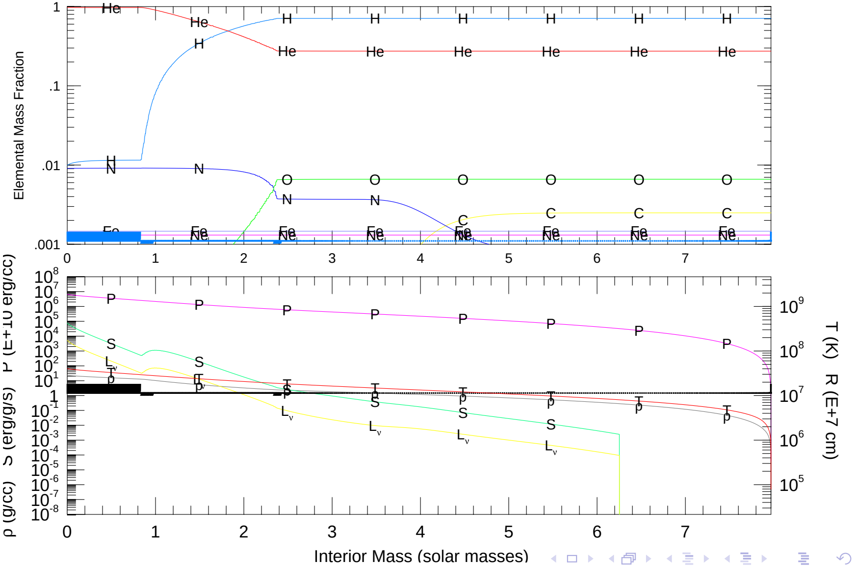
Kippenhahn Diagram, 8 $M_\odot$ Star



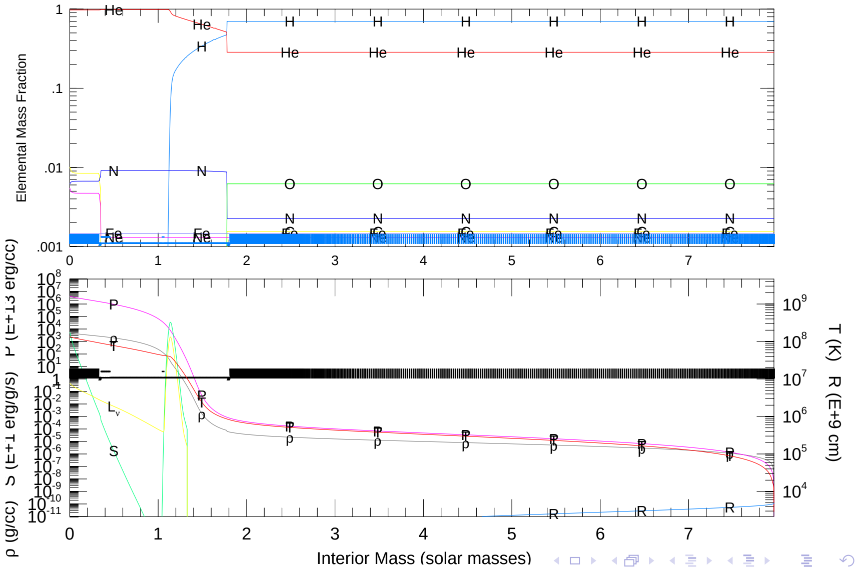
8 M$_\odot$ Star, Middle of Hydrogen Burning



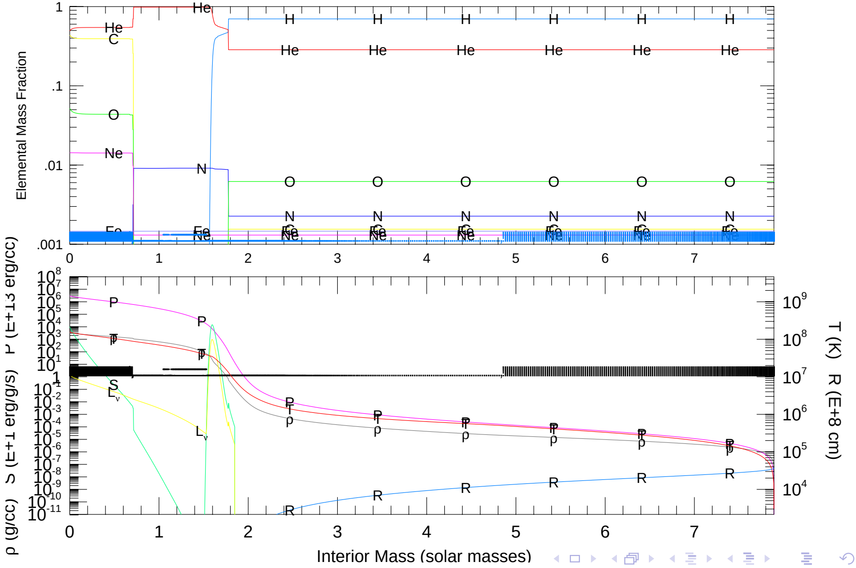
8 $M_\odot$ Star, End of Hydrogen Burning



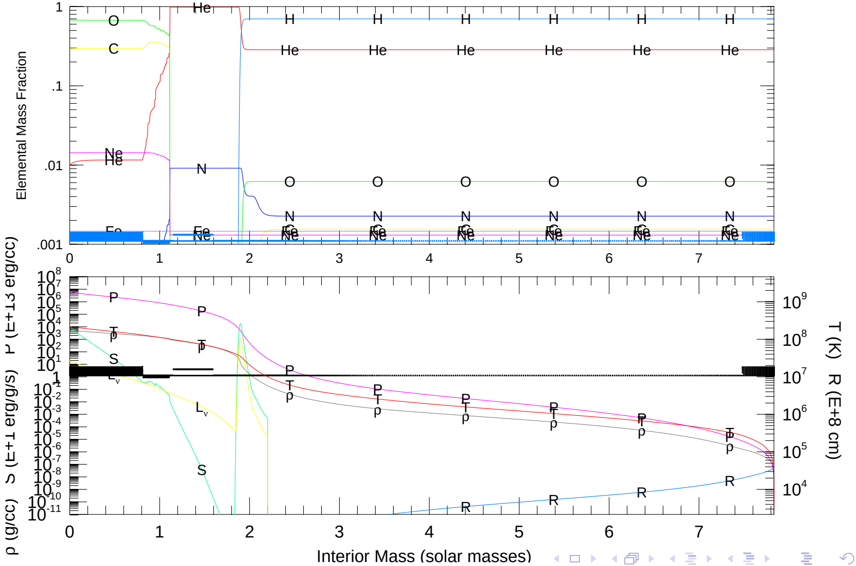
8 $M_\odot$ Star, Beginning of Helium Burning



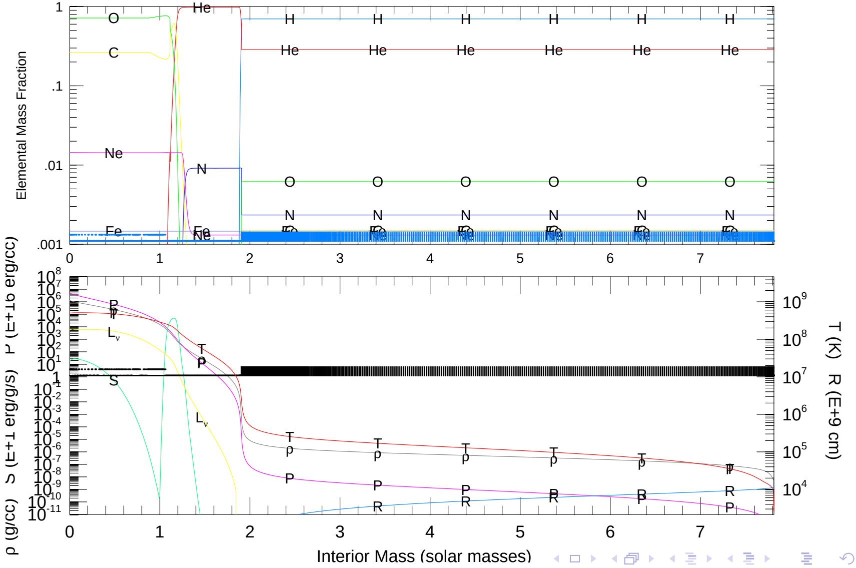
8 $M_\odot$ Star, Middle of Helium Burning



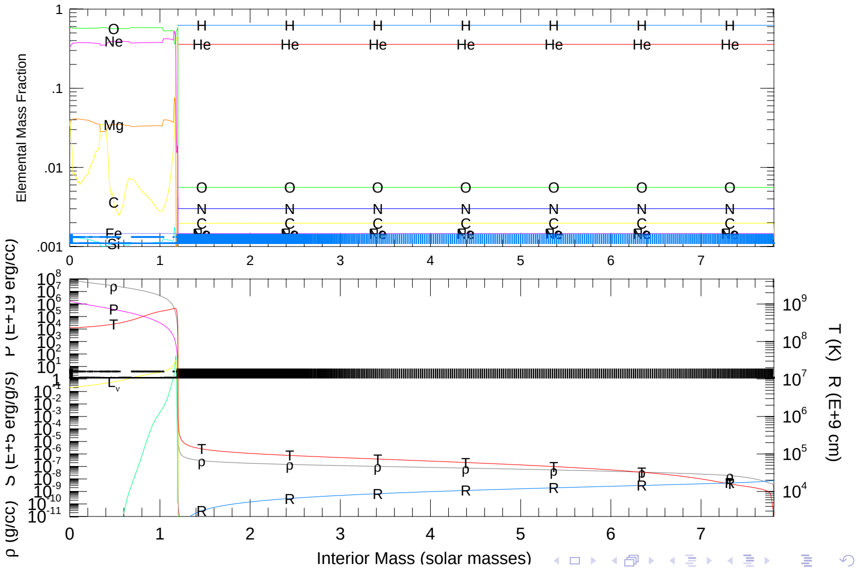
8 $M_\odot$ Star, End of Helium Burning



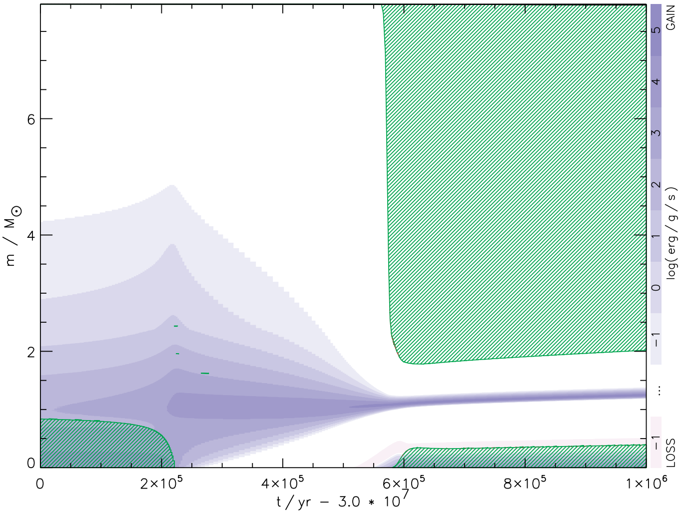
8 $M_\odot$ Star, $Tc = 5\times10^8$ K



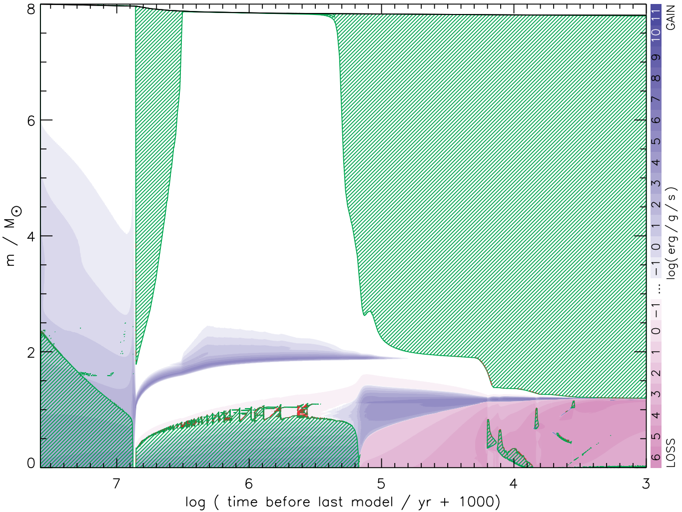
8 $M_\odot$ Star, Last Model Computed



Kippenhahn Diagram, $8\,M_\odot$ Star, He Ignition



Kippenhahn Diagram, $8\,M_\odot$ Star



Kippenhahn Diagram, 8 $M_\odot$ Star, Off-Center C Ignition