Astrophysics I: Stars and Stellar Evolution

AST 4001

Yong-Zhong Qian[1]

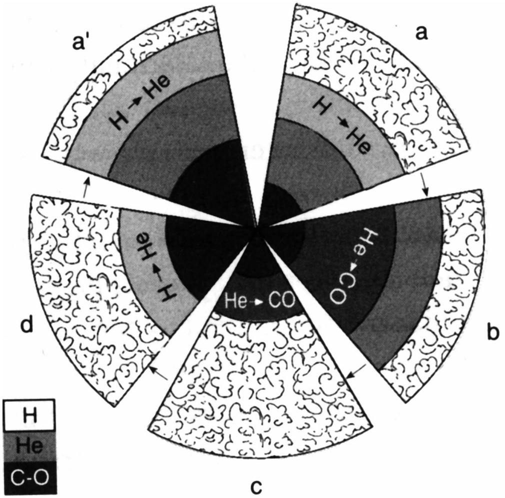
[1]School of Physics and Astronomy
University of Minnesota

Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - Planetary Nebula Formation
2. White Dwarf Stars
   - White Dwarf Star Formation
   - White Dwarf Star Structure
   - White Dwarf Star Evolution

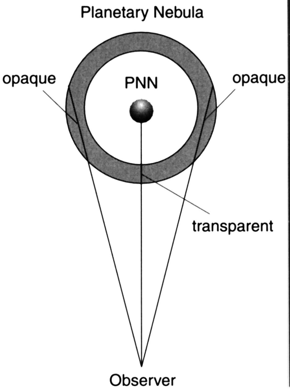
Change of Structure During AGB Cycle

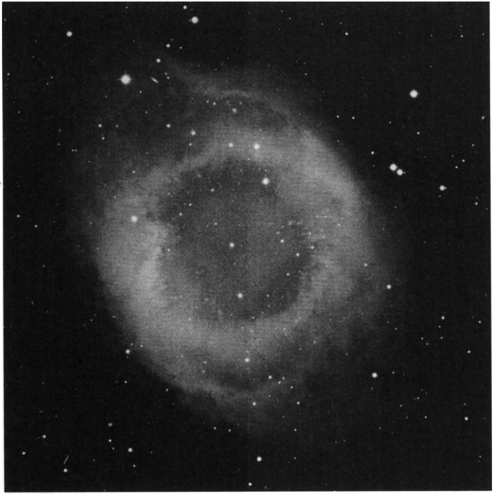


Planetary Nebula (PN) Formation

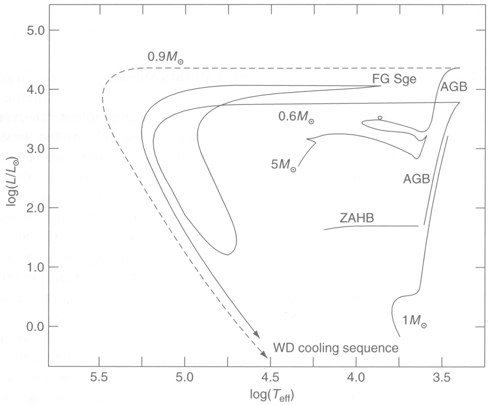
- eventually all extended AGB red giant envelope is lost by stellar winds
- star gets very hot ($30,000\,\mathrm{K}$)
- blows a fast hot superwind that evacuates bubble around it
- formation of "ring"-shaped "planetary" nebula
- eventually burning in shell stops when mass of shell drops below $10^{-3}\ldots10^{-4}\,\mathrm{M}_\odot$
- lower mass stars make lower mass WDs
- more lower mass stars, therefore more lower-mass remnants

Planetary Nebula Ring





Evolution of Intermediate and Low-Mass Stars in the HRD



- evolution of low-mass (1 $M_\odot$) and intermediate-mass (5 $M_\odot$) star in the HRD
- low-mass star ascends Hayashi line, then "jumps" to zero-age horizontal branch (ZAHB) at He ignition, becomes AGB.
- in this example, a late He flash after most envelope lost brings stars back to red before it descends to WD branch

Overview

1. Recap
   - Planetary Nebula Formation
2. White Dwarf Stars
   - White Dwarf Star Formation
   - White Dwarf Star Structure
   - White Dwarf Star Evolution

White Dwarfs from AGB - Surface Composition

- during Asymptotic Giant Branch (AGB) cycle of double-shell hydrogen/helium burning
- eventually all envelope lost due to stellar winds
- star descends from AGB, superwind $\Rightarrow$ Planetary Nebula + White Dwarf
- depending on where in AGB cycle envelope is lost - H shell of He shell burning - we find H or He surface layers
- since He shell phase is much shorter, the star is less likely in that phase
- *observationally* we find that about 25 % of WDs have He envelope, 75 % have H envelope
- ONeMg white dwarfs from the high-mass end of intermediate mass stars, CO white dwarfs for lower masses

Helium White Dwarfs

- Stars of $\sim 0.7\,\mathrm{M}_\odot - 1\,\mathrm{M}_\odot$ lose *entire* envelope in the Red Giant Phase before igniting central helium burning
- $\Rightarrow$ no AGB phase, no PN phase
- $\Rightarrow$ Helium white dwarf stars
- typical white dwarf star masses: $\sim 0.2 - 0.4\,\mathrm{M}_\odot$
- NOTE:
  recall that stars with initial mass below $0.7\,\mathrm{M}_\odot$ live longer than that current age of the universe
  $\Rightarrow$ no white dwarfs from such stars

White Dwarf Star Masses

Stars with initial masses $\sim 7.5 - 9\,\mathrm{M}_\odot$

- exceed CO core mass of $\sim 1.1\,\mathrm{M}_\odot$
- ignite central carbon burning $\Rightarrow$ make ONeMg core
- do not ignite later burning stages
- lose envelope as AGB stars (+PN)
- $\Rightarrow$ ONeMg WDs with $M > 1.1\,\mathrm{M}_\odot$
- but due to IMF: few stars with $7.5\,\mathrm{M}_\odot < M < 9\,\mathrm{M}_\odot$

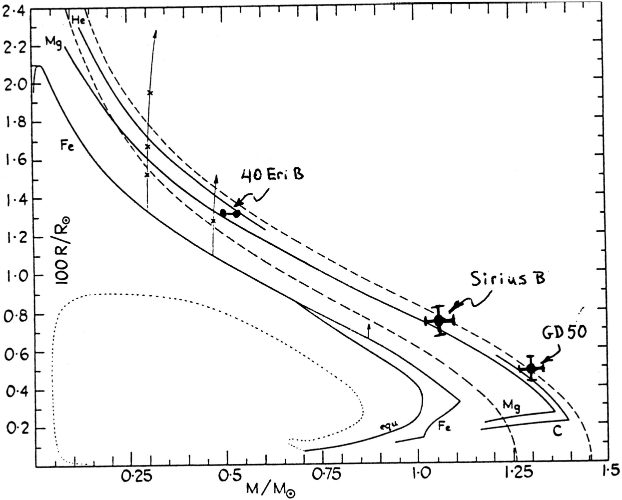
White Dwarf Star Masses

- stars with initial mass $2\,M_\odot \lesssim M \lesssim 7.5\,M_\odot$:
  - non-degenerate ignition of central He burning $\Rightarrow$ CO core
  - no ignition of carbon burning $\Rightarrow$ CO WD
  - make range of WD with mass below $\sim 1.1\,M_\odot$
  - IMF $\Rightarrow$ more stars
- stars with initial mass $1\,M_\odot \lesssim M \lesssim 2\,M_\odot$:
  - formation degenerate He core, ignites when grown to $\sim 0.6\,M_\odot$
  - $\Rightarrow$ CO Core
  - no ignition of carbon burning $\Rightarrow$ CO WD of about that mass
  - IMF $\Rightarrow$ many stars

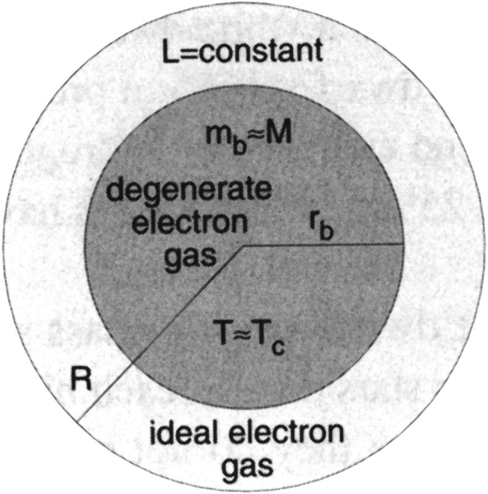
White Dwarf Star Masses

- stars with initial mass $0.7\,\mathrm{M}_\odot \lesssim M \lesssim 1\,\mathrm{M}_\odot$
  - do not ignite carbon burning
  - $\Rightarrow$ He WD
  - typical masses: $\sim 0.2 - 0.4\,\mathrm{M}_\odot$
  - IMF $\Rightarrow$ many stars
- in binary star system
  - stellar core may be uncovered due to loss of envelope by interaction with companion star
  - $\Rightarrow$ typically occurs when star expands
  - $\Rightarrow$ at beginning RG or AGB phases
  - $\Rightarrow$ different mass distribution, typically lower masses

Mass-Radius Relation for Degenerate Stars



White Dwarf Structure



Structure of a WD (sketch)

- degenerate core with constant temperature, T_c, radius r_b, and mass r_b
- non-degenerate envelope with constant luminosity
- white dwarf radius R

White Dwarf Structure

approximate picture:

- degenerate electron gas is good heat conductor $\Rightarrow$
- assume core of mass $r_\mathrm{b} \approx M$, of constant $T \approx T_\mathrm{c}$, and radius r_b
- envelope has small mass, no energy generation, ideal gas $\Rightarrow$ constant luminosity $F = L$
- for the envelope the structure equations reduce do

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho\frac{GM}{r^2}\,, \qquad \frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2}$$

- assume power-law Kramers opacity for envelope

$$\kappa = \kappa_0 \rho T^{-7/2} = \kappa_0 \frac{\mu}{\mathcal{R}} P T^{-9/2}$$

White Dwarf Structure

- combining the three equations we obtain

$$P\,\mathrm{d}P = \frac{16\pi a c \mathcal{R} G}{3\kappa_0\mu}\frac{M}{L}T^{15/2}\,\mathrm{d}T$$

- integrating inward from the surface where $P = 0 = T$ we obtain

$$P(T) = \left(\frac{64\pi a c \mathcal{R} G}{51\kappa_0\mu}\right)^{1/2}\left(\frac{M}{L}\right)^{1/2}T^{17/4}$$

- Using the ideal gas equation we may rewrite this in the form

$$\rho(T) = \left(\frac{64\pi a c \mu G}{51\kappa_0\mathcal{R}}\right)^{1/2}\left(\frac{M}{L}\right)^{1/2}T^{13/4}$$

White Dwarf Structure

- let us define the transition from non-degenerate envelope to degenerate core as the location where ideal and (completely) degenerate electron gas pressure become equal to define location r_b:

$$\left[\mathcal{R}\frac{\rho}{\mu_\mathrm{e}}T\right]_\mathrm{b} = \left[K_1'\left(\frac{\rho}{\mu_\mathrm{e}}\right)^{5/3}\right]_\mathrm{b}, \quad \rho(r_\mathrm{b}) = \mu_\mathrm{e}\left(\frac{\mathcal{R}T(r_\mathrm{b})}{K_1'}\right)^{3/2}$$

- at this location the envelope temperature must fit the core temperature, T_c, and the density should be continuous
- combining with the previous equation we obtain

$$\frac{L}{M} = C_\mathrm{WD}\,T_\mathrm{c}^{7/2}, \quad C_\mathrm{WD} = \frac{64\pi acG{K_1'}^3\mu}{51\mathcal{R}^4\kappa_0{\mu_\mathrm{e}}^2}$$

White Dwarf Structure

- for typical composition we obtain for luminosity

$$L \approx 6.8\times10^{-3}\left(\frac{T_{\mathrm{c}}}{10^7\,\mathrm{K}}\right)^{7/2}\left(\frac{M}{\mathrm{M}_\odot}\right)\mathrm{L}_\odot$$

- and for central temperature

$$T_{\mathrm{c}} \approx 4\times10^7\left(\frac{L}{M}\frac{\mathrm{M}_\odot}{\mathrm{L}_\odot}\right)^{2/7}\,\mathrm{K}$$

Quiz

- Is the white dwarf in hydrostatic equilibrium?
- Is the white dwarf in thermal equilibrium?
- Is the white dwarf in nuclear equilibrium?
- The white dwarf shines.
  What is the energy source?
- Try to find the answers yourself (2 min).
- Discuss with your neighbor(s) and write down your concordance solution. (2 min)
- Open discussion. (2 min)

White Dwarf Cooling

- Luminosity of white dwarf from internal energy of the ions (ideal gas)

$$U_\mathrm{I} = \frac{3}{2}\frac{\mathcal{R}}{\mu_\mathrm{I}} M T_\mathrm{c}$$

- $\Rightarrow$ luminosity equals loss in internal energy

$$L = -\frac{\mathrm{d}U_\mathrm{I}}{\mathrm{d}t} = -\frac{3}{2}\frac{\mathcal{R}}{\mu_\mathrm{I}} M \frac{\mathrm{d}T_\mathrm{c}}{\mathrm{d}t}$$

- using $L \propto T_\mathrm{c}^{7/2}$ (recall $L = M\,C_\mathrm{WD}\,T_\mathrm{c}^{7/2}$) we can write

$$L = -\frac{3}{7}\frac{\mathcal{R}}{\mu_\mathrm{I}} M \frac{T_\mathrm{c}}{L}\frac{\mathrm{d}L}{\mathrm{d}t}, \qquad \frac{\mathrm{d}L}{\mathrm{d}t} = -\frac{7}{3}\frac{\mu_\mathrm{I}}{\mathcal{R}}\frac{L^2}{T_\mathrm{c} M}$$

- using $L = M\,C_\mathrm{WD}\,T_\mathrm{c}^{7/2}$ again, we eliminate L and obtain

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -\frac{7}{3}\frac{\mu_\mathrm{I}}{\mathcal{R}} M\,{C_\mathrm{WD}}^2\,T_\mathrm{c}^6$$

White Dwarf Cooling Time

- define cooling time as time scale of luminosity drop (by e):

$$\tau_{\mathrm{cool}} = -\frac{\mathrm{d}t}{\mathrm{d}\ln L} = -L\left(\frac{\mathrm{d}L}{\mathrm{d}t}\right)^{-1} = \frac{3}{7}\frac{\mathcal{R}}{\mu_{\mathrm{I}}}\frac{M}{L}T_{\mathrm{c}}$$

- eliminating T_{c} using $L/M = C_{\mathrm{WD}}\,T_{\mathrm{c}}^{7/2}$ we obtain

$$\tau_{\mathrm{cool}} = \frac{3}{7}\frac{\mathcal{R}}{\mu_{\mathrm{I}}\,{C_{\mathrm{WD}}}^{2/7}}\left(\frac{M}{L}\right)^{5/7} \approx 2.5\times10^{6}\left(\frac{M}{\mathrm{M}_{\odot}}\right)^{5/7}\left(\frac{L}{\mathrm{L}_{\odot}}\right)^{-5/7}\mathrm{yr}$$

- e.g., cooling time scale for $1\,\mathrm{M}_{\odot}$ WD at (to)
  $10^{-1}\,\mathrm{L}_{\odot}$: $\sim 10^{7}\,\mathrm{yr}$
  $10^{-4}\,\mathrm{L}_{\odot}$: $\sim 2\times10^{9}\,\mathrm{yr}$

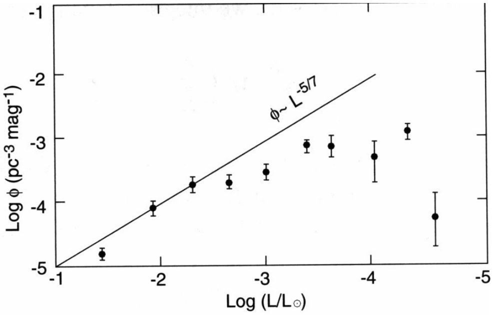
White Dwarf Cooling Time

NOTES:

- white dwarf quickly "out-shined" by surrounding planetary nebula ($L = 10^4\,\mathrm{L}_\odot$)
- when $L \sim 10^{-4}$ is reached, T_c bcomes low enough for crystallization to set in
    - first rise in heat capacity from $3/2k_\mathrm{B}/\mathrm{ion}$ to $3k_\mathrm{B}/\mathrm{ion}$
    - then, however, heat capacity drops quickly $\propto T^3$ (below Debye Temperature)
    - $\Rightarrow$ fast drop in WD luminosity
    - $\Rightarrow$ fewer WDs in given L bin
- radius of WD essentially constant
  $\Rightarrow$ WDs of given mass follow track of constant radius

$$\log L = 4\log T_\mathrm{eff} + 2\log R + \log(4\pi\sigma)$$

White Dwarf Luminosity Function



- considering the white dwarf cooling function

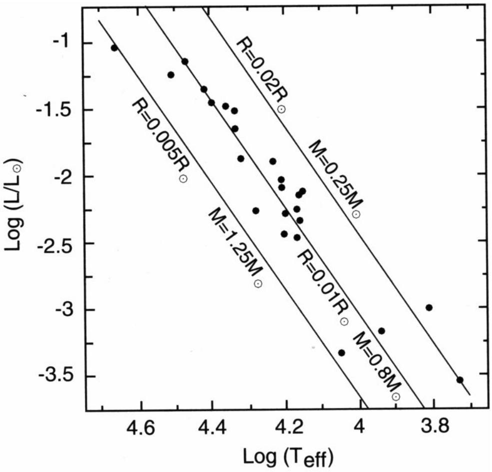
$$\tau_{\rm cool} \propto L^{-5/7}$$

- $\Rightarrow$ number of stars within given luminosity bin

$$\Phi \propto L^{-5/7}$$

- drop-off below $L \sim 10^{-4}\,\mathrm{L}_\odot$ due to crystallization

White Dwarf H-R Diagram



white dwarfs follow temperature-luminosity relation for constant radius:

$$\log L = 4 \log T_{\mathrm{eff}} + $$

$$+ 2 \log R + \log (4\pi\sigma)$$

White Dwarf Spectra

Spectral Type	Characteristics
DA	Balmer Lines only: no He I or metals present
DB	He I lines (4026Å, 4471Å, 4713Å) : no H or metals present
DO	He II lines (4686Å)
DZ	Metal lines only (CaII, Fe, O): no H or He
DQ	Carbon features, C_2
DC	Continuous spectrum; no lines

White Dwarf Birth and Evolution in HRD