Astrophysics I: Stars and Stellar Evolution
AST 4001

Alexander Heger[1,2,3]

[1]School of Physics and Astronomy
University of Minnesota

[2]Nuclear & Particle Physics, Astrophysics & Cosmology Group, T-2
Los Alamos National Laboratory

[3]Department of Astronomy and Astrophysics
University of California at Santa Cruz

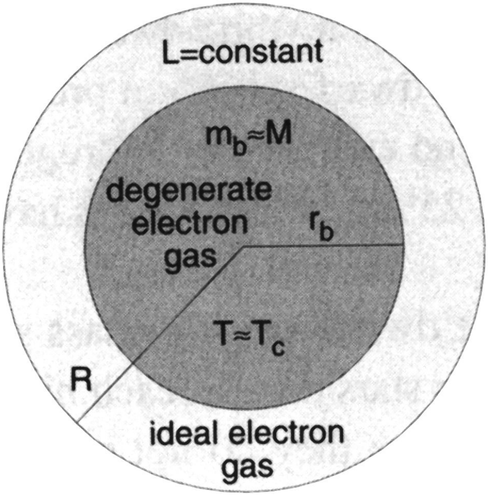
Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - White Dwarf Star Structure

2. White Dwarfs
   - White Dwarf Star Evolution
   - White Dwarfs in the HRD

White Dwarf Structure



Structure of a WD (sketch)

- degenerate core with constant temperature, T_c, radius r_b, and mass r_b
- non-degenerate envelope with constant luminosity
- white dwarf radius R

White Dwarf Structure

approximate picture:

- degenerate electron gas is good heat conductor $\Rightarrow$
- assume core of mass $r_{\mathrm{b}} \approx M$, of constant $T \approx T_{\mathrm{c}}$, and radius r_{b}
- envelope has small mass, no energy generation, ideal gas $\Rightarrow$ constant luminosity $F = L$
- for the envelope the structure equations reduce do

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho\frac{GM}{r^2}\,, \quad \frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2}$$

- assume power-law Kramers opacity for envelope

$$\kappa = \kappa_0\rho T^{-7/2} = \kappa_0\frac{\mu}{\mathcal{R}}PT^{-9/2}$$

White Dwarf Structure

- combining the three equations we obtain

$$P\,\mathrm{d}P = \frac{16\pi ac\mathcal{R}G}{3\kappa_0\mu}\frac{M}{L}T^{15/2}\,\mathrm{d}T$$

- integrating inward from the surface where $P = 0 = T$ we obtain

$$P(T) = \left(\frac{64\pi ac\mathcal{R}G}{51\kappa_0\mu}\right)^{1/2}\left(\frac{M}{L}\right)^{1/2}T^{17/4}$$

- Using the ideal gas equation we may rewrite this in the form

$$\rho(T) = \left(\frac{64\pi ac\mu G}{51\kappa_0\mathcal{R}}\right)^{1/2}\left(\frac{M}{L}\right)^{1/2}T^{13/4}$$

White Dwarf Structure

- let us define the transition from non-degenerate envelope to degenerate core as the location where ideal and (completely) degenerate electron gas pressure become equal to define location r_b:

$$\left[\mathcal{R}\frac{\rho}{\mu_\mathrm{e}}T\right]_\mathrm{b} = \left[K_1'\left(\frac{\rho}{\mu_\mathrm{e}}\right)^{5/3}\right]_\mathrm{b}\,, \quad \rho(r_\mathrm{b}) = \mu_\mathrm{e}\left(\frac{\mathcal{R}T(r_\mathrm{b})}{K_1'}\right)^{3/2}$$

- at this location the envelope temperature must fit the core temperature, T_c, and the density should be continuous
- combining with the previous equation we obtain

$$\frac{L}{M} = C_\mathrm{WD}\,T_\mathrm{c}^{7/2}\,, \quad C_\mathrm{WD} = \frac{64\pi acG{K_1'}^3\mu}{51\mathcal{R}^4\kappa_0{\mu_\mathrm{e}}^2}$$

White Dwarf Structure

- for typical composition we obtain for luminosity

$$L \approx 6.8\times10^{-3}\left(\frac{T_\mathrm{c}}{10^7\,\mathrm{K}}\right)^{7/2}\left(\frac{M}{\mathrm{M}_\odot}\right)\mathrm{L}_\odot$$

- and for central temperature

$$T_\mathrm{c} \approx 4\times10^7\left(\frac{L}{M}\frac{\mathrm{M}_\odot}{\mathrm{L}_\odot}\right)^{2/7}\mathrm{K}$$

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Quiz

- Is the white dwarf in hydrostatic equilibrium?
- Is the white dwarf in thermal equilibrium?
- Is the white dwarf in nuclear equilibrium?
- The white dwarf shines.
  What is the energy source?
- Try to find the answers yourself (2 min).
- Discuss with your neighbor(s) and write down your concordance solution. (2 min)
- Open discussion. (2 min)

White Dwarf Cooling

- Luminosity of white dwarf from internal energy of the ions (ideal gas)

$$U_{\mathrm{I}} = \frac{3}{2}\frac{\mathcal{R}}{\mu_{\mathrm{I}}} M T_{\mathrm{c}}$$

- $\Rightarrow$ luminosity equals loss in internal energy

$$L = -\frac{\mathrm{d}U_{\mathrm{I}}}{\mathrm{d}t} = -\frac{3}{2}\frac{\mathcal{R}}{\mu_{\mathrm{I}}} M \frac{\mathrm{d}T_{\mathrm{c}}}{\mathrm{d}t}$$

- using $L \propto T_{\mathrm{c}}^{7/2}$ (recall $L = M\,C_{\mathrm{WD}}\,T_{\mathrm{c}}^{7/2}$) we can write

$$L = -\frac{3}{7}\frac{\mathcal{R}}{\mu_{\mathrm{I}}} M \frac{T_{\mathrm{c}}}{L}\frac{\mathrm{d}L}{\mathrm{d}t}\,, \qquad \frac{\mathrm{d}L}{\mathrm{d}t} = -\frac{7}{3}\frac{\mu_{\mathrm{I}}}{\mathcal{R}}\frac{L^2}{T_{\mathrm{c}} M}$$

- using $L = M\,C_{\mathrm{WD}}\,T_{\mathrm{c}}^{7/2}$ again, we eliminate L and obtain

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -\frac{7}{3}\frac{\mu_{\mathrm{I}}}{\mathcal{R}} M {C_{\mathrm{WD}}}^2 T_{\mathrm{c}}^6$$

White Dwarf Cooling Time

- define cooling time as time scale of luminosity drop (by e):

$$\tau_{\mathrm{cool}} = -\frac{\mathrm{d}t}{\mathrm{d}\ln L} = -L\left(\frac{\mathrm{d}L}{\mathrm{d}t}\right)^{-1} = \frac{3}{7}\frac{\mathcal{R}}{\mu_{\mathrm{I}}}\frac{M}{L}T_{\mathrm{c}}$$

- eliminating T_{c} using $L/M = C_{\mathrm{WD}}\,T_{\mathrm{c}}^{7/2}$ we obtain

$$\tau_{\mathrm{cool}} = \frac{3}{7}\frac{\mathcal{R}}{\mu_{\mathrm{I}}\,{C_{\mathrm{WD}}}^{2/7}}\left(\frac{M}{L}\right)^{5/7} \approx 2.5\times10^{6}\left(\frac{M}{\mathrm{M}_{\odot}}\right)^{5/7}\left(\frac{L}{\mathrm{L}_{\odot}}\right)^{-5/7}\mathrm{yr}$$

- e.g., cooling time scale for $1\,\mathrm{M}_{\odot}$ WD at (to)
  $10^{-1}\,\mathrm{L}_{\odot}$: $\sim 10^{7}\,\mathrm{yr}$
  $10^{-4}\,\mathrm{L}_{\odot}$: $\sim 2\times10^{9}\,\mathrm{yr}$

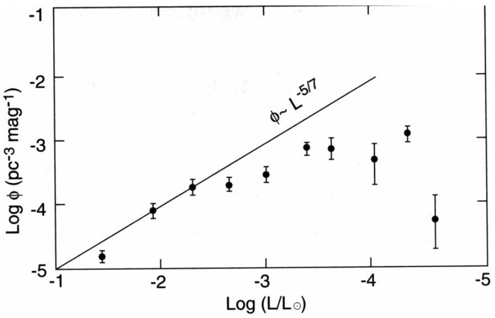
White Dwarf Cooling Time

NOTES:

- white dwarf quickly "out-shined" by surrounding planetary nebula ($L = 10^4\,\mathrm{L}_\odot$)
- when $L \sim 10^{-4}$ is reached, T_c bcomes low enough for crystallization to set in
  - first rise in heat capacity from $3/2k_\mathrm{B}/\mathrm{ion}$ to $3k_\mathrm{B}/\mathrm{ion}$
  - then, however, heat capacity drops quickly $\propto T^3$ (below Debye Temperature)
  - $\Rightarrow$ fast drop in WD luminosity
  - $\Rightarrow$ fewer WDs in given L bin
- radius of WD essentially constant
  $\Rightarrow$ WDs of given mass follow track of constant radius

$$\log L = 4 \log T_\mathrm{eff} + 2 \log R + \log (4\pi\sigma)$$

White Dwarf Luminosity Function



- considering the white dwarf cooling function

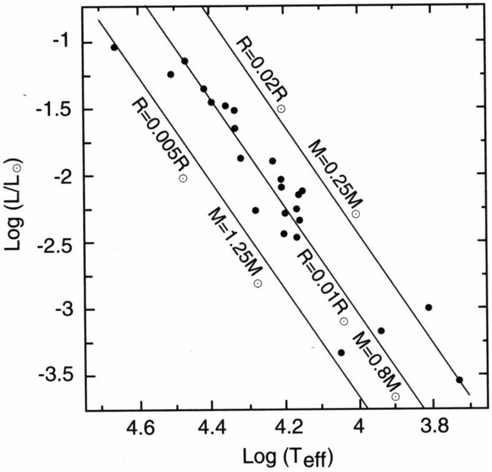
$$\tau_\mathrm{cool} \propto L^{-5/7}$$

- $\Rightarrow$ number of stars within given luminosity bin

$$\Phi \propto L^{-5/7}$$

- drop-off below $L \sim 10^{-4}\,\mathrm{L}_\odot$ due to crystallization

White Dwarf H-R Diagram



white dwarfs follow temperature-luminosity relation for constant radius:

$$\log L = 4 \log T_{\mathrm{eff}} +$$

$$+2 \log R + \log\left(4\pi\sigma\right)$$

White Dwarf Spectra

Spectral Type	Characteristics
DA	Balmer Lines only: no He I or metals present
DB	He I lines (4026Å, 4471Å, 4713Å) : no H or metals present
DO	He II lines (4686Å)
DZ	Metal lines only (CaII, Fe, O): no H or He
DQ	Carbon features, C_2
DC	Continuous spectrum; no lines

White Dwarf Birth and Evolution in HRD