Astrophysics I: Stars and Stellar Evolution
AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - Neutron Stars
   - Neutron Star Structure
   - Pulsars

2. Black Holes
   - Formation of Black Holes
   - Schwarzschild Radius
   - Kerr Black Holes

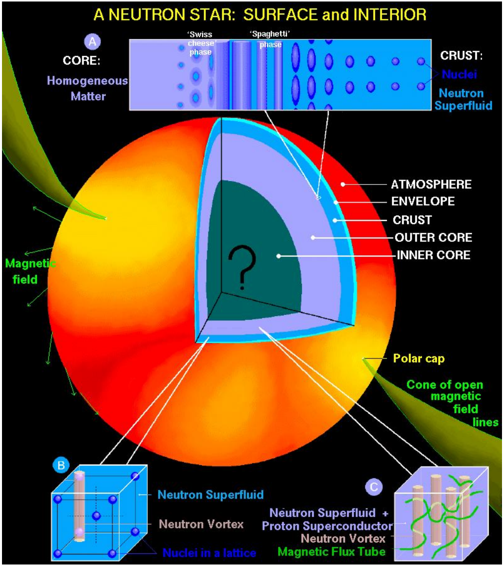
Overview - Comparison of Compact Remnants

Distinguishing Traits of Compact Objects

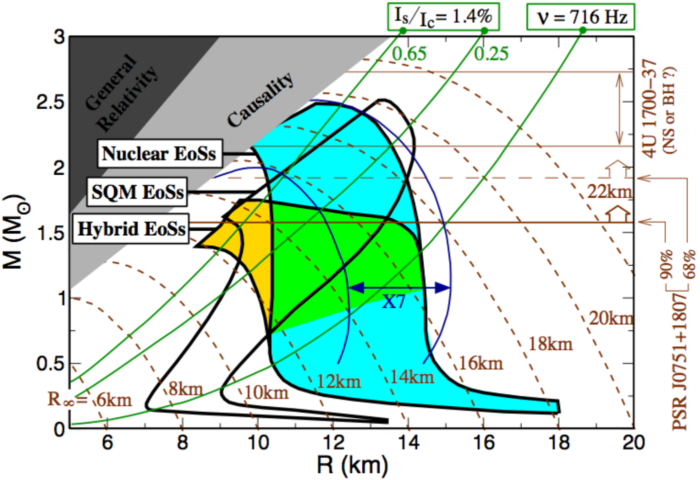
Object	Mass[a] (M)	Radius[b] (R)	Mean Density (g cm^{-3})	Surface Potential (GM/Rc^2)
Sun	$M_\odot$	$R_\odot$	1	10^{-6}
White dwarf	$\lesssim M_\odot$	$\sim 10^{-2} R_\odot$	$\lesssim 10^7$	$\sim 10^{-4}$
Neutron star	$\sim 1\text{–}3 M_\odot$	$\sim 10^{-5} R_\odot$	$\lesssim 10^{15}$	$\sim 10^{-1}$
Black hole	Arbitrary	$2GM/c^2$	$\sim M/R^3$	~ 1

[a] $M_\odot = 1.989 \times 10^{33}$ g
[b] $R_\odot = 6.9599 \times 10^{10}$ cm

Neutron Star Structure



Neutron Star Equation(s) of State



Neutron Stars

- end point of evolution of (some) massive stars
- remain after supernova
- masses are $\sim 1.2 \ldots 2 \mathrm{M}_{\odot}$, radii around 10 km
- maximum mass determined by uncertain neutron star and nuclear equation of state
- minimum mass determined by stellar evolution
- rotating neutron stars with magnetic fields appear as pulsars

Neutron Star Mass Determination in Binaries

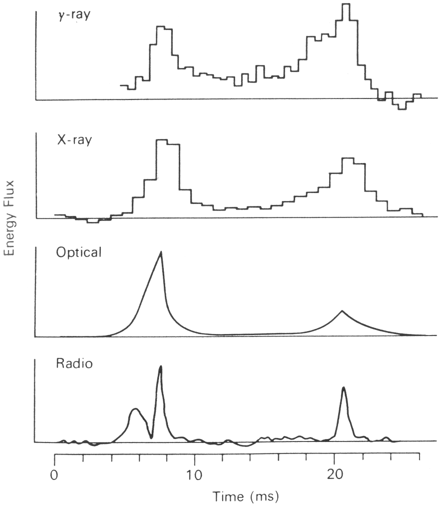
- assume system with two stars of
  masses M_1 and M_2,
  velocities v_1 and v_2,
  distances from center of mass a_1 and a_2
- doppler shift due to orbital motion
  if seen at inclination angle i is

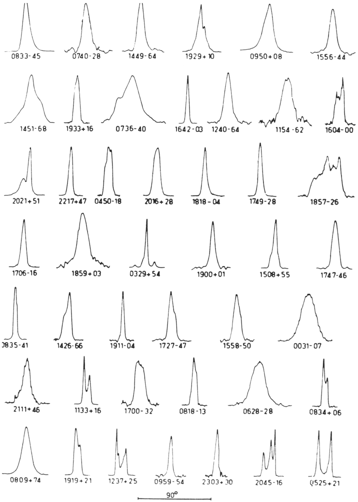
$$v_1 = \frac{2\pi}{P}\, a_1 \sin i$$

- mass function

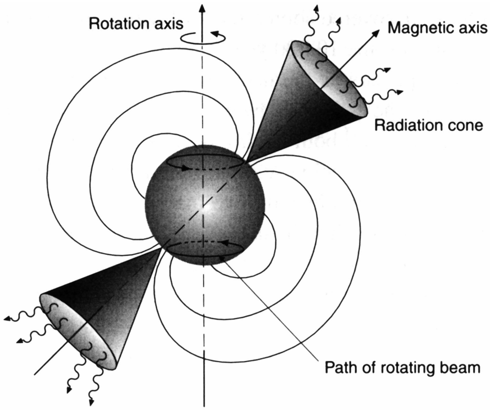
$$f(M_1, M_2, i) = \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2} = \frac{P\, v_1}{2\pi G}$$

Pulsar Radiation - Energy Bands, Zoo





Pulsars



moment of interia

$$I = \frac{8\pi}{3} \int_0^R r^2 \rho(r) \, \mathrm{d}r$$

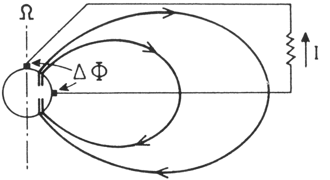
dipol radiation

$$L \sim \frac{1}{6c^3} B^2 R^6 \Omega^4$$

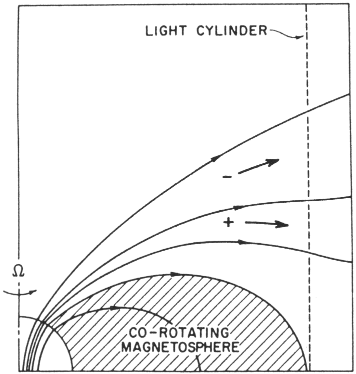
spindown

$$L = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2} I \Omega^2\right)$$

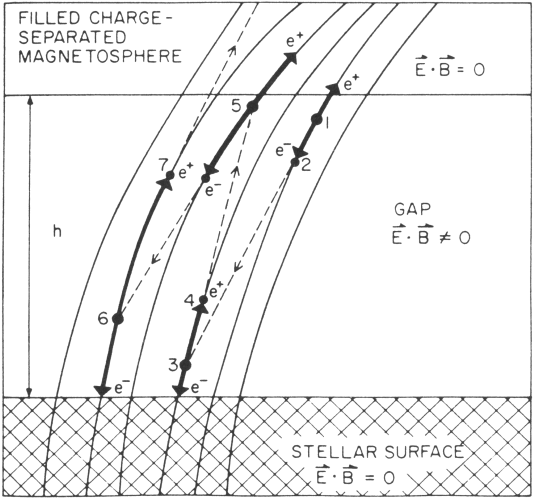
Pulsar - Magnets



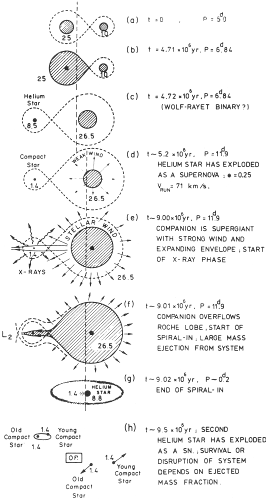
	IRON MAGNET	NEUTRON STAR
R (cm)	10	10^{6}
P (sec)	0.015	1
B (Gauss)	10^{4}	10^{12}
$\Delta\Phi$ (Volts)	5	3×10^{16}

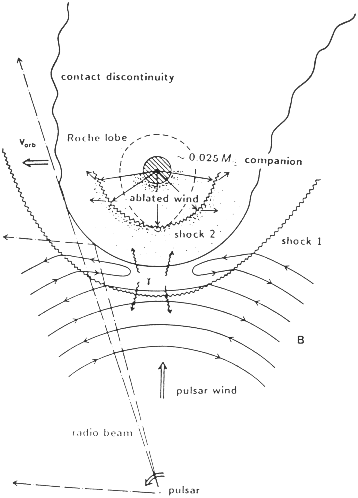


Pulsar - Magnetosphere

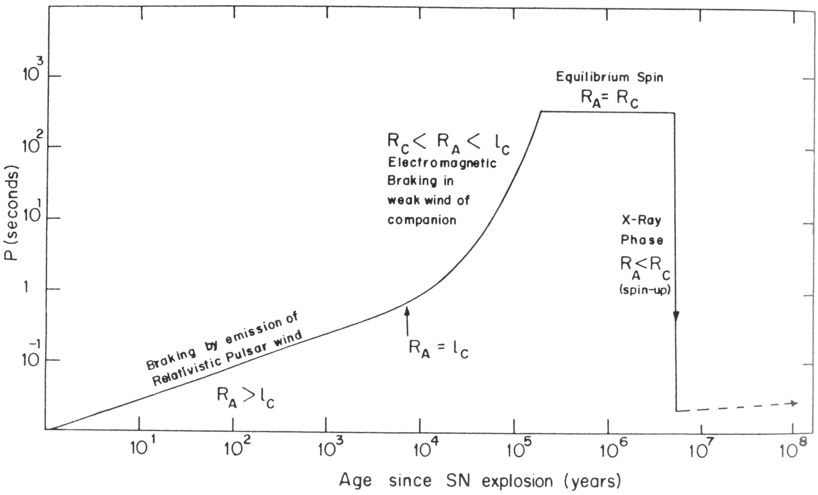


Pulsar Evolution - Binary Star System





Pulsar Evolution - Spindown in Binary Systems



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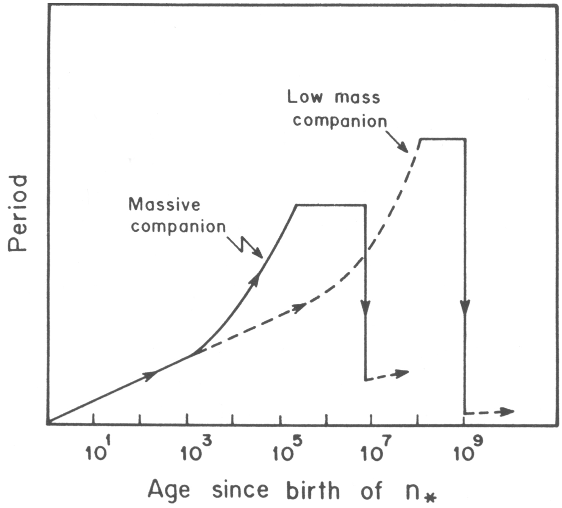
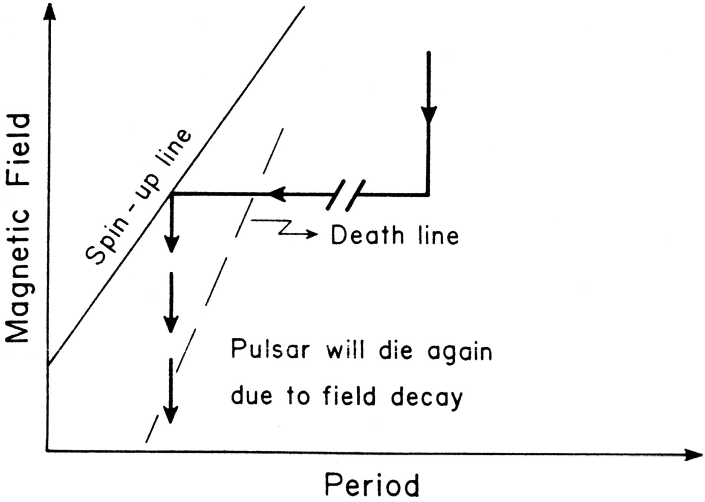
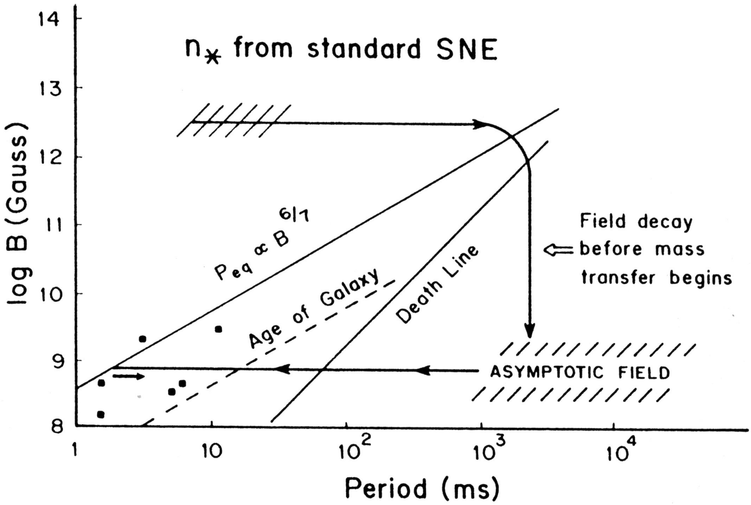


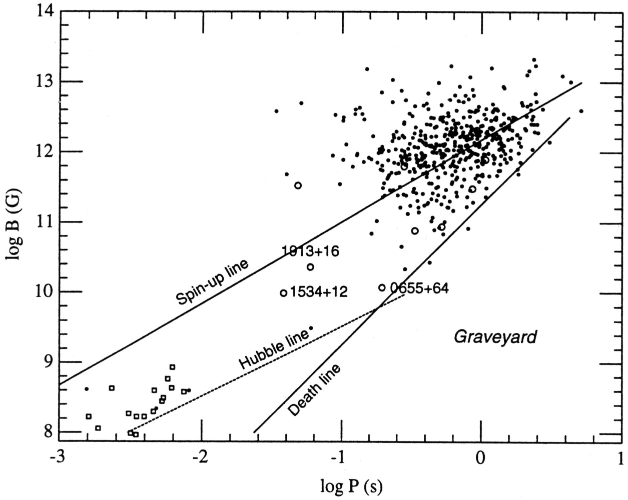
Pulsar Evolution - Spinup Due to Accretion



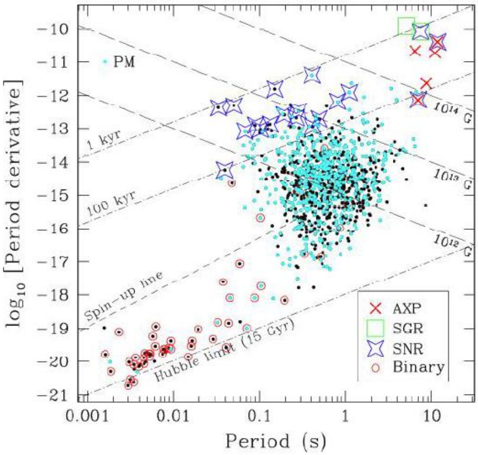
Pulsar Evolution



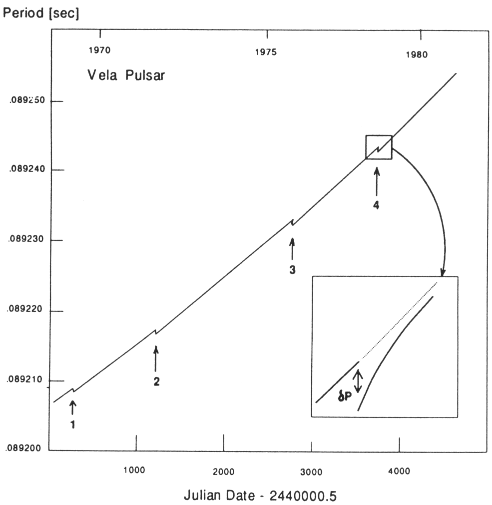
Observed Pulsar Populations



Pulsar Period and Spin-down



Pulsar Glitches



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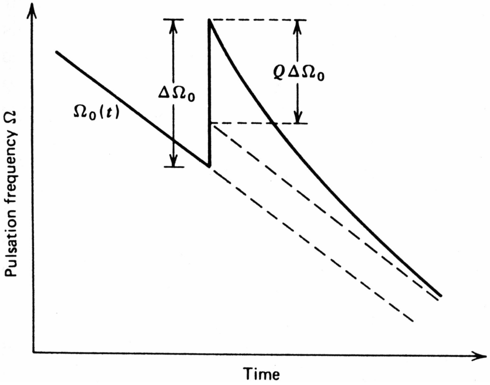

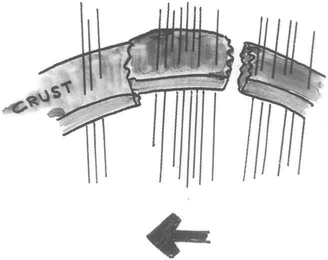

Change in rotation rate

$$\frac{\Delta\Omega}{\Omega} \sim 10^{-6}$$

change in spin-down rate

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} \sim 10^{-2}$$

Summary Pulsar

Two classes of pulsars

- young pulsars with strong magnetic fields and spin acquired during formation in supernova
- "recycled" millisecond pulsars spun up by accretion in binary star systems; weak magnetic fields.

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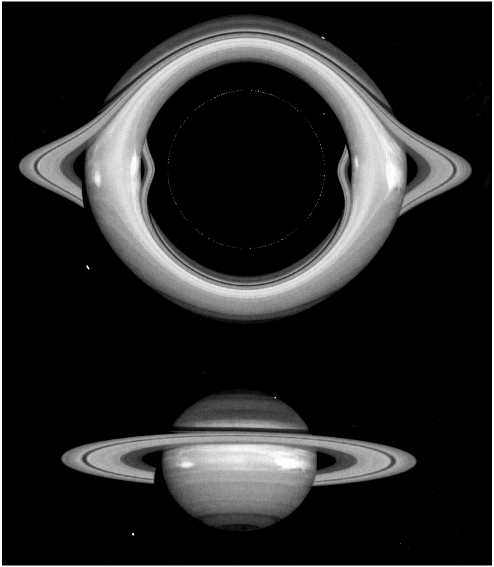
Black Holes



Saturn as seen through the gravitational lense of a black hole

Black Hole Formation

- beyond a critical mass repulsive nuclear forces can no longer hold up collapse against increasing gravity
- General Relativity make gravitational acceleration even stronger
- internal energy and pressure contribute to gravity
- black hole can form by accretion or directly during massive star collapse

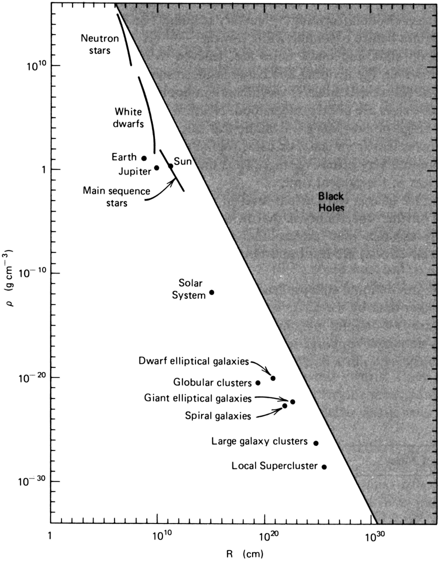
Schwarzschild Radius

- Schwarzschild Radius $R_s = 2GM/c^2$
- from inside Schwarzschild radius nothing can escape
  "**black**" holes
- rotating "Kerr" black holes have smaller Schwarzschild Radius
- Kerr black holes drag space time with them
- Kerr black holes have "egosphere"
  no counter-rotation/orbit possible
- spin parameter (specific angular momentum)

$$a = J/M\,, \quad 0 \le a \le GM/c$$

- maximum spin $\propto M$
- radius of extreme Kerr black hole: $R(a = GM/c) = R_s/2$

Density and Radii of Astronomical Objects



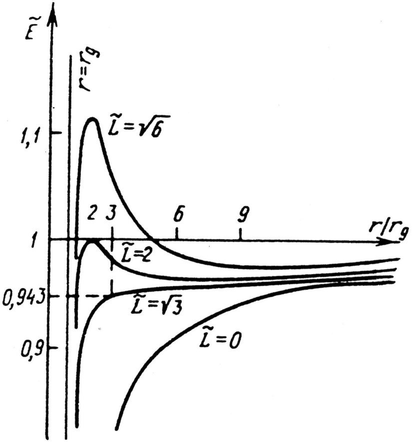
- comparison of average density of astronomical objects
- average density of black holes decreases with increasing mass

$$R_s \sim M$$

$$V \sim R^3$$

$$\bar{\rho} = M/V \sim M^{-2}$$

Black Holes - Angular Momentum and Orbits



in this figure the following conventions were used:

- "gravitational radius"
  $r_{\mathrm{g}} = R_{\mathrm{s}}$
- normalized angular momentum
  $\tilde{L} = l/cmr_{\mathrm{g}}$ where l is the angular momentum of the particle
- specific *total* energy of the particle
  $\tilde{E} = E/mc^2$
  $E = mc^2 + E_{\mathrm{bind}} + E_{\mathrm{kin}}$

Black Holes - Energy and Orbits

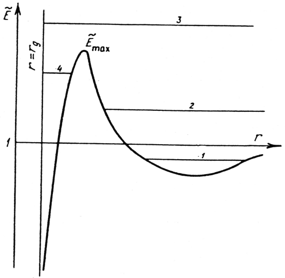


Fig. 3. Effective black hole potential. $1 - \tilde{E} = \tilde{E}_1$, $2 - \tilde{E} = \tilde{E}_2$, $3 - \tilde{E} = \tilde{E}_3$, $4 - \tilde{E} = \tilde{E}_4$

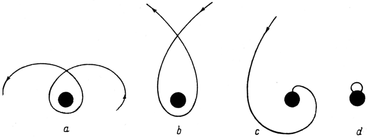


Fig. 4. Trajectories of particles with energies (a) $\tilde{E}_1$, (b) $\tilde{E}_2$, (c) $\tilde{E}_3$, and (d) $\tilde{E}_4$

Types of orbits

- **1/a**: bound/closed, $0 < \tilde{E} < \tilde{E}_{\rm max}$
- **2/b**: unbound/open, $\tilde{E} < 0$
- **3/c**: "unbound"/capture for $\tilde{E} > \tilde{E}_{\rm max} > 0$ (not in classical mechanics)
- **4/d**: closed capture loop $\tilde{E} < \tilde{E}_{\rm max}$, can be $\tilde{E} < 0$ or $\tilde{E} > 0$ (not in classical mechanics)

Kerr Black Holes

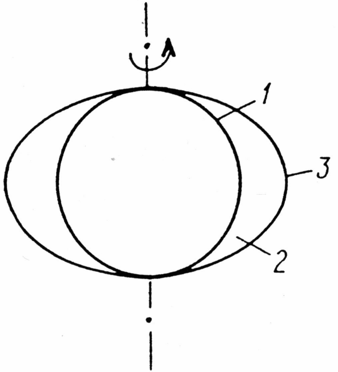
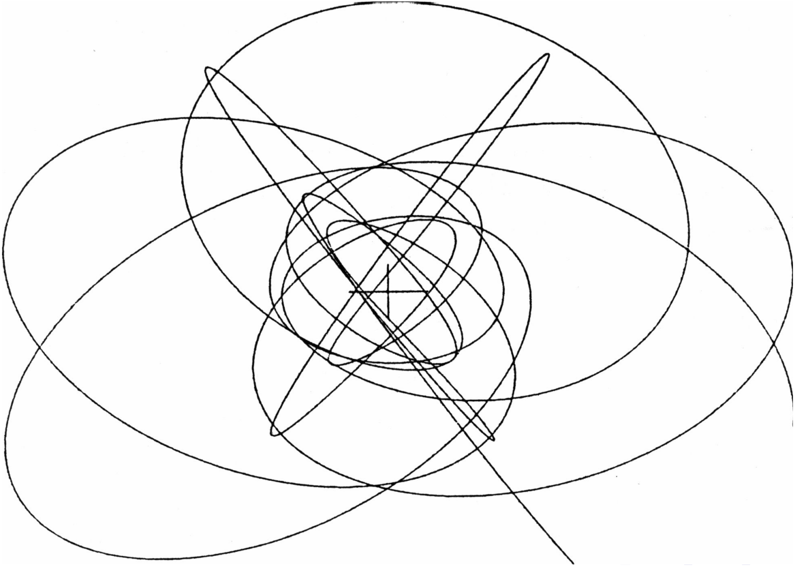


Fig. 7. A rotating black hole: 1—horizon, 2—ergosphere, 3—static limit

Particle Orbit around a Kerr Black Hole



Critical Radii in Black Holes

Orbit	$a = 0$	$a = M$	
		$L > 0$	$L < 0$
r_{photon}	1.5	0.5	2.0
r_{bind}	2.0	0.5	2.92
r_{bound}	3.0	0.5	4.5

Schwarzschild Case

- photon circular orbit

$$r_{\mathrm{bind}} = 1.5 R_{\mathrm{s}} = 3\frac{GM}{c^2}$$

- last stable orbit

$$r_{\mathrm{bound}} = 3 R_{\mathrm{s}} = 6\frac{GM}{c^2}\,,\ v = \frac{c}{2}$$

- last marginally stable circular orbit

$$r_{\mathrm{bind}} = 2 R_{\mathrm{s}} = 4\frac{GM}{c^2}\,,\ v = v_{\mathrm{esc}}$$

Energy of last Stable Orbit

- Schwarzschild black hole

$$E_{\text{bind}} = 0.057 mc^2$$

$\Rightarrow$

$$L_{\text{acc}} = 0.057 \dot{M} c^2 \approx 3\times 10^{36} \left(\frac{\dot{M}}{10^{-9} \mathrm{M}_\odot/\mathrm{yr}} \right) \mathrm{erg\,s}^{-1}$$

- Kerr black hole

$$E_{\text{bind}} = 0.42 mc^2$$

$\Rightarrow$

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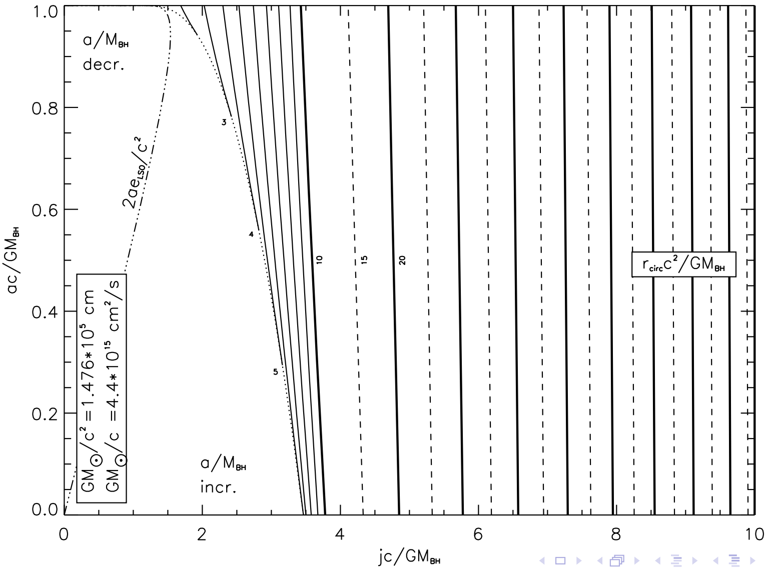
Orbits and Energies



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