

Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

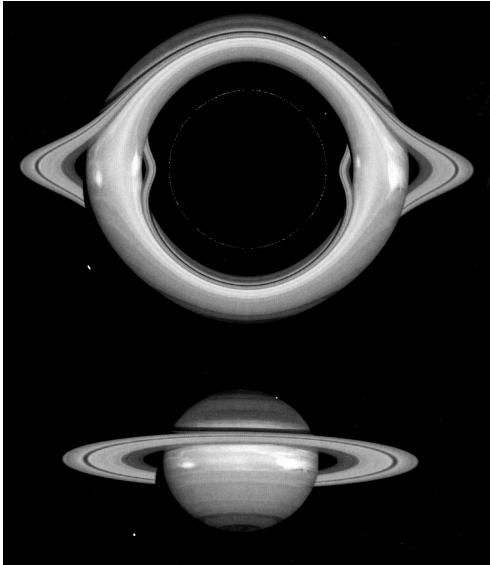
1 Recap

- Formation of Black Holes
- Schwarzschild Radius
- Kerr Black Holes

2 Supernovae

- Things that blow up
- Pair-Instability Supernovae

Black Holes



Saturn as seen through
the gravitational lense of a
black hole

Black Hole Formation

- beyond a critical mass repulsive nuclear forces can no longer hold up collapse against increasing gravity
- **General Relativity** make gravitational acceleration even stronger
- internal energy and pressure contribute to gravity
- black hole can form by accretion or directly during massive star collapse

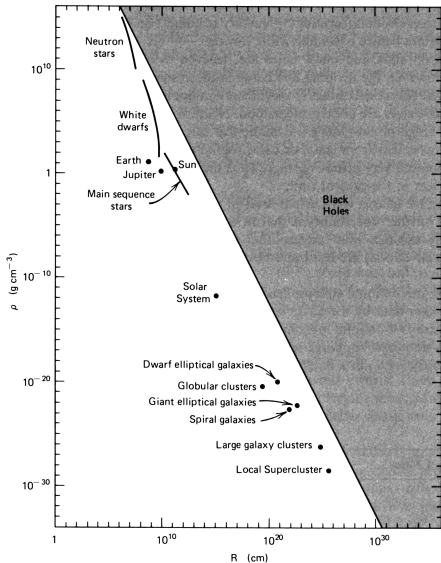
Schwarzschild Radius

- Schwarzschild Radius $R_s = 2GM/c^2$
- from inside Schwarzschild radius nothing can escape “black” holes
- rotating “Kerr” black holes have smaller Schwarzschild Radius
- Kerr black holes drag space time with them
- Kerr black holes have “egosphere”
no counter-rotation/orbit possible
- spin parameter (specific angular momentum)

$$a = J/M, \quad 0 \leq a \leq GM/c$$

- maximum spin $\propto M$
- radius of extreme Kerr black hole: $R(a = GM/c) = R_s/2$

Density and Radii of Astronomical Objects



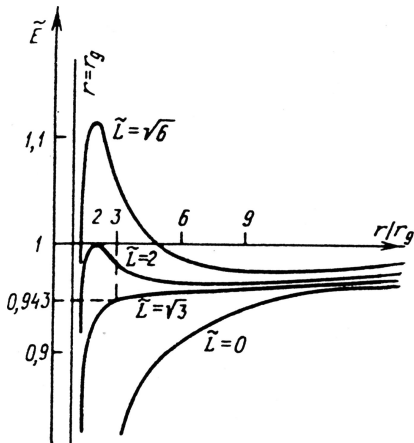
- comparison of average density of astronomical objects
- average density of black holes decreases with increasing mass

$$R_S \sim M$$

$$V \sim R^3$$

$$\bar{\rho} = M/V \sim M^{-2}$$

Black Holes - Angular Momentum and Orbits



in this figure the following conventions were used:

- “gravitational radius”
 $r_g = R_s$
- normalized angular momentum
 $\tilde{L} = l/cmr_g$ where l is the angular momentum of the particle
- specific *total* energy of the particle
 $\tilde{E} = E/mc^2$
 $E = mc^2 + E_{\text{bind}} + E_{\text{kin}}$

Black Holes - Energy and Orbits

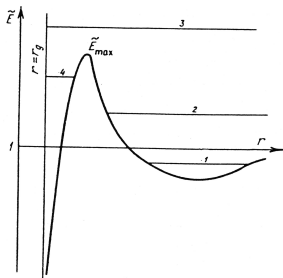


Fig. 3. Effective black hole potential. 1 - $\tilde{E} = \tilde{E}_1$, 2 - $\tilde{E} = \tilde{E}_2$, 3 - $\tilde{E} = \tilde{E}_3$, 4 - $\tilde{E} = \tilde{E}_4$

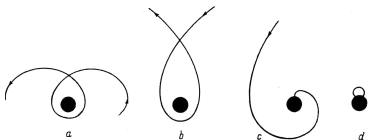


Fig. 4. Trajectories of particles with energies (a) \tilde{E}_1 , (b) \tilde{E}_2 , (c) \tilde{E}_3 , and (d) \tilde{E}_4

Types of orbits

- **1/a:** bound/closed,
 $0 < \tilde{E} < \tilde{E}_{max}$
- **2/b:** unbound/open, $\tilde{E} < 0$
- **3/c:** “unbound” /capture for
 $\tilde{E} > \tilde{E}_{max} > 0$
(not in classical mechanics)
- **4/d:** closed capture loop
 $\tilde{E} < \tilde{E}_{max}$,
can be $\tilde{E} < 0$ or $\tilde{E} > 0$
(not in classical mechanics)

Kerr Black Holes

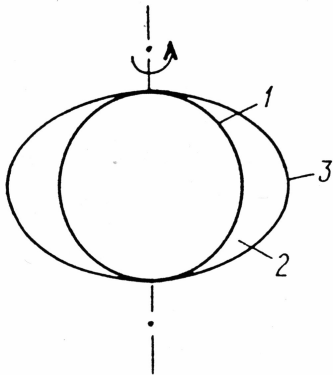
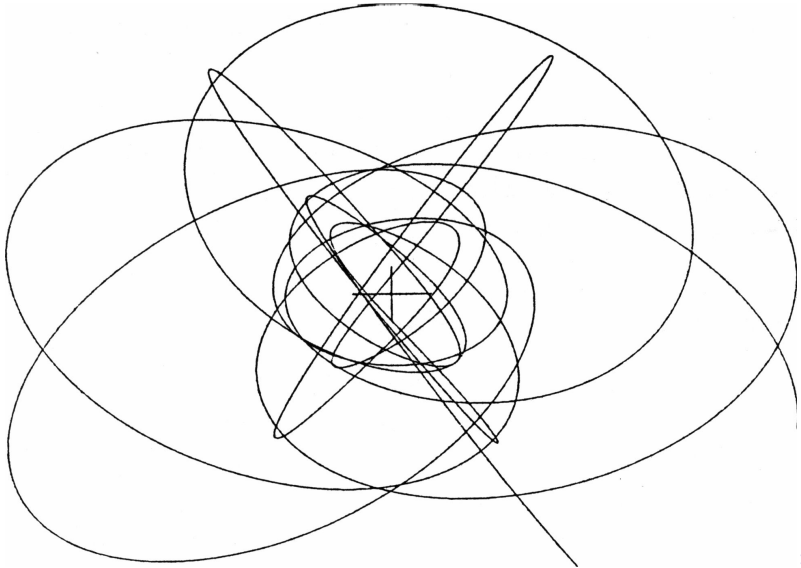


Fig. 7. A rotating black hole: 1—horizon, 2—ergosphere, 3—static limit

Particle Orbit around a Kerr Black Hole



Critical Radii in Black Holes

Schwarzschild Case

- photon circular orbit

$$r_{\text{bind}} = 1.5R_s = 3\frac{GM}{c^2}$$

- last stable orbit

$$r_{\text{bound}} = 3R_s = 6\frac{GM}{c^2}, v = \frac{c}{2}$$

- last marginally stable circular orbit

$$r_{\text{bind}} = 2R_s = 4\frac{GM}{c^2}, v = v_{\text{esc}}$$

Orbit	$a = 0$	$a = M$	
		$L > 0$	$L < 0$
r_{photon}	1.5	0.5	2.0
r_{bind}	2.0	0.5	2.92
r_{bound}	3.0	0.5	4.5

Energy of last Stable Orbit

- Schwarzschild black hole

$$E_{\text{bind}} = 0.057mc^2$$

⇒

$$L_{\text{acc}} = 0.057\dot{M}c^2 \approx 3 \times 10^{36} \left(\frac{\dot{M}}{10^{-9}M_{\odot}/\text{yr}} \right) \text{erg s}^{-1}$$

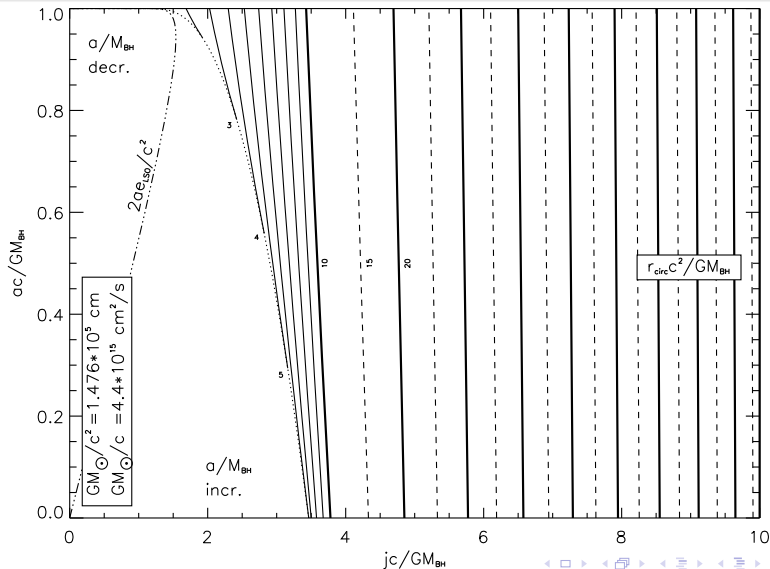
- Kerr black hole

$$E_{\text{bind}} = 0.42mc^2$$

⇒

$$L_{\text{acc}} = 0.42\dot{M}c^2 \approx 3 \times 10^{37} \left(\frac{\dot{M}}{10^{-9}M_{\odot}/\text{yr}} \right) \text{erg s}^{-1}$$

Orbits and Energies



Summary - Comparison of Compact Remnants

Distinguishing Traits of Compact Objects

Object	Mass ^a (M)	Radius ^b (R)	Mean Density (g cm^{-3})	Surface Potential (GM/Rc^2)
Sun	M_{\odot}	R_{\odot}	1	10^{-6}
White dwarf	$\lesssim M_{\odot}$	$\sim 10^{-2} R_{\odot}$	$\lesssim 10^7$	$\sim 10^{-4}$
Neutron star	$\sim 1-3 M_{\odot}$	$\sim 10^{-5} R_{\odot}$	$\lesssim 10^{15}$	$\sim 10^{-1}$
Black hole	Arbitrary	$2GM/c^2$	$\sim M/R^3$	~ 1

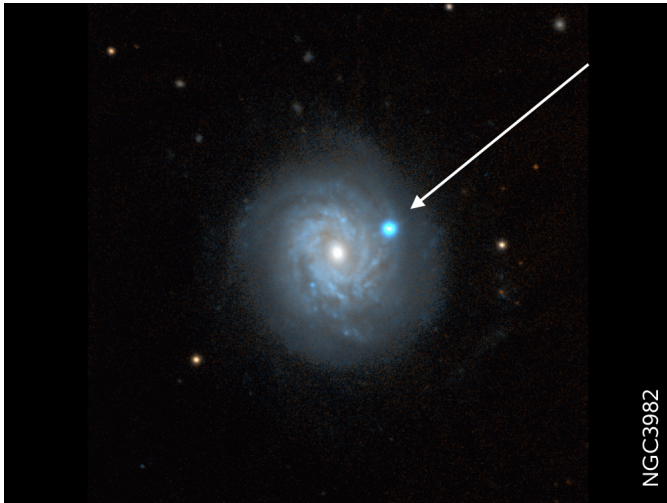
$${}^a M_{\odot} = 1.989 \times 10^{33} \text{ g}$$

$${}^b R_{\odot} = 6.9599 \times 10^{10} \text{ cm}$$

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Supernovae



Supernovae - Overview

Things that blow up

supernovae from massive stars

- CO white dwarf \rightarrow Type Ia SN, $E \approx 1$ Bethe
- MgNeO WD, accretion \rightarrow AIC, faint SN
- “SAGB” star (AGB, then SN) \rightarrow EC SN
- “normal” SN (Fe core collapse) \rightarrow Type II SN
- WR star (Fe CC) \rightarrow Type Ib/c
- “Collapsar”, GRB \rightarrow broad line Ib/a SN, “hypernova”
- Pulsational pair SN \rightarrow multiple, nested Type I/II SN
- Very massive stars \rightarrow pair SN, $\lesssim 100B$ ($1B=10^{51}$ erg)
- Very massive collapsar \rightarrow IMBH, SN, hard transient
- Supermassive stars \rightarrow $\gtrsim 100000 B$ SN or SMBH



$1B=10^{51}$ erg

MASS
 \downarrow

Supernovae

Things that blow up

Neutron star-powered supernovae

- CO white dwarf → Type Ia SN, $E \approx 1$ Bethe
- MgNeO WD, accretion → AIC, faint SN
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Supernovae

Things that blow up

Thermonuclear supernovae (no r -process)

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Supernovae

Things that blow up

Black hole-powered supernovae (“Collapsars”)

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Pair-Instability Supernovae

