Astrophysics I: Stars and Stellar Evolution

AST 4001

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Stars and Stellar Evolution, Fall 2008

Overview

1. Recap
   - The Magnitude
   - Filter and Colors
   - Absolute Magnitude and Distance Modulus
2. Luminosity Classes and Spectral Types
   - Spectral Type
   - Luminosity Class
   - Limits to the Planck Function

Introducing - The Magnitude

- Basic definition of magnitude from relative brightness of stars.
- The difference in magnitude between two stars, **1** and **2**, with apparent brightnesses (as we see/observe) them in the sky, s_1 and s_2, is given by

$$m_1 - m_2 = -2.5 \log \left(\frac{s_1}{s_2} \right)$$

- If a star at distance r has a surface luminosity of L_λ in wavelength λ a radiation flow of

$$f_\lambda = \frac{L_\lambda}{4\pi r^2}$$

reaches us.

The Magnitude

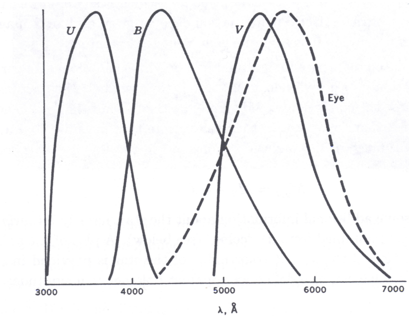
- If our instrument, filer, or eye has a response function E_λ we obtain

$$s = \frac{1}{4\pi r^2} \int_0^\infty L_\lambda E_\lambda \mathrm{d}\lambda$$

- The magnitude is then given, up to a constant, by

$$m = -2.5 \log \frac{1}{4\pi r^2} \int_0^\infty L_\lambda E_\lambda \mathrm{d}\lambda \quad + \text{const.}$$

Filters, Color Index



Filter Functions, arbitrary units (should be normalized to 1)

- We define a color index by $m_x - m_y$ for filters x and y by

$$\text{color index} = m_x - m_y$$

- x, y is, e.g., "U", "B", "V" for ultraviolet, blue, and visible.
- They have centers at $\lambda_{\mathrm{U}} = 365\,\mathrm{nm}$, $\lambda_{\mathrm{B}} = 440\,\mathrm{nm}$, and $\lambda_{\mathrm{V}} = 584\,\mathrm{nm}$.

Absolute Magnitude, Distance Modulus

- We often define
  $U = m_\mathrm{B}$, $B = m_\mathrm{B}$, and $V = m_\mathrm{B}$, etc.
- We define the absolute magnitude M of a star by its apparent magnitude if it were at a distance of 10 pc
  (1 pc = 3.26 light years)
- With this definition we obtain the distance modulus by

$$m - M = 5 \log \left(\frac{r}{10\,\mathrm{pc}} \right)$$

- The bolometric luminosity is the total luminosity of a star through its entire spectrum, i.e., $E_\lambda = 1$.

Absolute Bolometric Magnitude

- The apparent bolometric luminosity of the sun is $m_{\mathrm{bol},\odot} = -26.83\,\mathrm{Mag}$
- The bolometric correction, B.C., is defined by B.C. $= m_{\mathrm{B}} - m_{\mathrm{bol}}$
- The absolute magnitude of the sun is $M_{\mathrm{bol},\odot} = 4.74\,\mathrm{Mag}$
- The absolute bolometric magnitude of a star is hence given by

$$M_{\mathrm{bol}} = 4.74 - 2.5\log\left(\frac{L}{\mathrm{L}_\odot}\right) \quad [\mathrm{Mag}]$$

Quiz

Starting from the definition of magnitude differences and the definition of the absolute magnitude, derive the formula for the distance modulus, $m - M$.

- $$m_1 - m_2 = -2.5 \log \left(\frac{s_1}{s_2} \right)$$
- We define the absolute magnitude M of a star by its apparent magnitude if it were at a distance of 10 pc

Instructions:

- Work on this yourself and write down your solution (2min)
- Discuss in groups of 2-3 (2min)

Quiz - Solution

Starting from the definition of magnitude differences and the definition of the absolute magnitude, derive the formula for the distance modulus, $m - M$.

Definition of magnitude differences:

$$m_1 - m_2 = -2.5 \log \left(\frac{s_1}{s_2} \right) \ ,$$

where s_1 and s_2 are the apparent brightnesses (as we see/observe) them in the sky. The *absolute magnitude* M is the apparent magnitude at a distance of 10 pc. Hence the *distance modulus*, $m - M$ is the is the difference in apparent magnitude of the star if it were at a distance of 10 pc to its true (observed) magnitude. We also use the flux, i.e., the observed brightness drops according to the inverse square law, $1/d^2$ where d is the distance, replacing m_1 by m, m_2 by M, s_1 by s and s_2 by $s \times (d/10\,\mathrm{pc})^2$ we obtain.

$$m - M = -2.5 \log \left(\frac{s}{s \times (d/10\,\mathrm{pc})^2} \right) = -2.5 \log \left(\left[\frac{10\,\mathrm{pc}}{d} \right]^2 \right) = 5 \log \left(\frac{d}{10\,\mathrm{pc}} \right)$$

NOTE: Here we have neglected the effects of extinction, filter functions, and bolometric corrections, etc.

Quiz

Compute the distance modulus of the sun.

Instructions:

- Work on this yourself and write down your solution (2min)
- Discuss in groups of 2-3 (2min)

Quiz - Solution

Compute the distance modulus of the sun.

The sun has a distance of one astronomical unit,
1.49598×10^{13} cm $= 4.84813681\times10^{-6}$ pc.
Hence we have

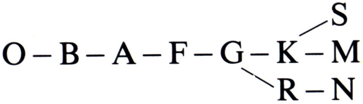
$$(m - M)_{\odot} = 5\log\left(\frac{4.84813681\times10^{-6}\,\mathrm{pc}}{10\,\mathrm{pc}}\right)$$

$$(m - M)_{\odot} = 5\log\left(4.84813681\times10^{-7}\right) = -31.57\,\mathrm{Mag}$$

Overview

Classification of Stars by Spectral Type

- We classify stars by their spectral type:



blue white red

- We use subtypes 0-9 with 0 being the hottest and 9 the coolest within each class.
- That is, O9 is followed by B0.

Naming Ionization Levels

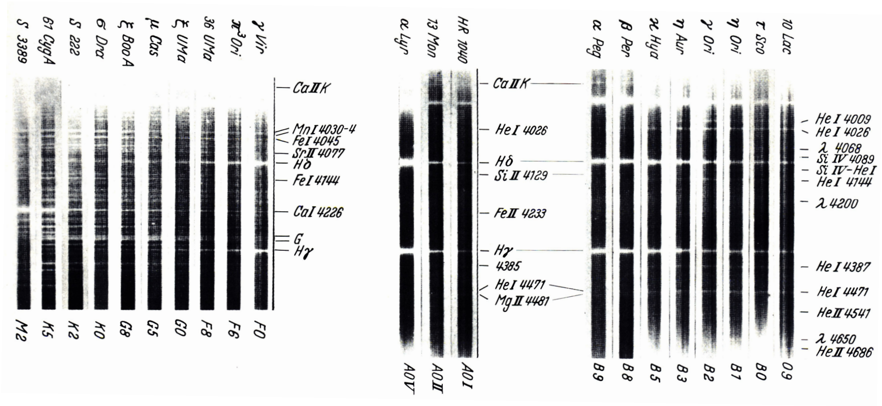
Ionization levels are named using capital roman letters:

- "I" corresponds to the neutral atom,
- "II" to the single ionized atom,
- "III" to the double ionized atom,
- and so forth.

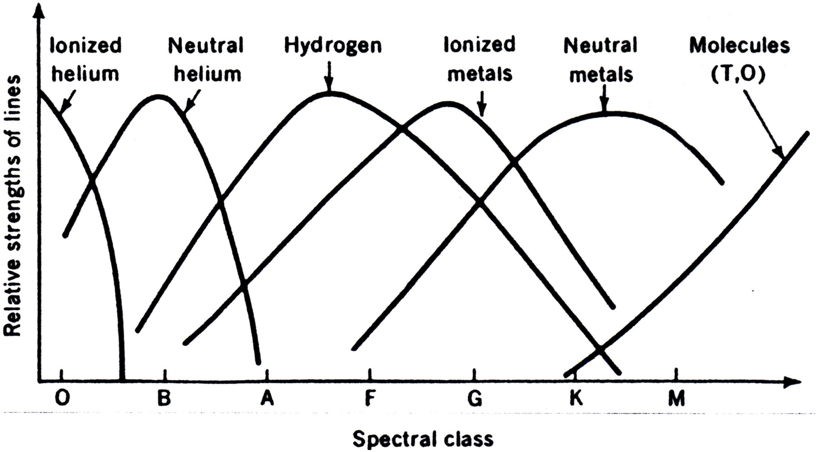
Examples:

- HeII: single ionized helium
- FeXXVII: completely ionized iron

Spectra - Classification



Ionization levels as a function of spectral class



Spectral Classification

O Blue stars with surface temperatures of 20,000 to 35,000 K. Spectra show multiple ionized atoms, especially He II, C III, N III, O III, Si V. He I is visible, but H I is weak.

B Blue–white stars with surface temperatures of about 15,000 K. He II lines have disappeared. He I is strongest at B2. H I lines getting stronger. O II, Si II, and Mg II lines are visible.

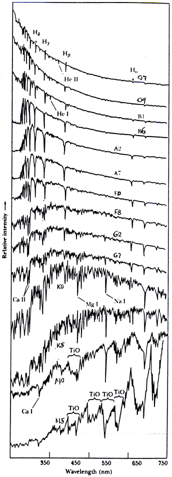
A White stars with surface temperatures of about 9000K. The H I lines dominate the spectrum and are strongest at A0. He I no longer visible. Neutral metal lines begin to appear.

F Yellow-white stars with surface temperatures of about 7000 K. The H I lines are getting weaker while Ca II are getting stronger. Many other metals such as Fe I, Fe II, Cr II, and Ti II are getting stronger.

G Yellow stars like the Sun with surface temperatures of about 5500 K. H I lines still getting weaker while Ca II lines are strongest at G0. Metal lines are getting stronger.

K Yellow-orange stars with surface temperatures of about 4000 K. Spectrum is dominated by metal lines. Ca I getting stronger. TiO bands become visible at K5.

M Red stars with surface temperatures of about 3000 K. TiO bands are very prominent. Ca I at 423 nm is very strong. Many neutral metal lines are seen. For stars cooler than M4, absorption by TiO is so severe that it is very difficult to find the continuum.



19-11 Spectral types Fourteen representative spectra are along with brief descriptions of the main features of each spectral ʼote that the overall shapes of the spectra, as well as the spectral lines, depend on the surface temperature of the stars. (Observations by G. Jacoby, D. Hunter, and C. Christian)

Class and Temperature

SPECTRAL TYPES AND TEMPERATURES
ON THE MAIN SEQUENCE

(mainly from Gray: The Observation and analysis of Stellar Photospheres 2nd ed., Cambridge Univ. Press)

O5	50,000	G5	5,680
O8	39,000	K0	5,270
B0	36,000	M0	4,045
B5	16,000	M5	3,000
B8	12,600	M8	2,200
A0	9,700	L0	2,000
A5	7,880	L8	1,500
F0	6,950	T0	1,300
F5	6,445	T8	800
G0	5,950		

Class and Temperature

O Blue stars with surface temperatures of about 20,000 – 35,000 K. Spectra show multiple ionized atoms, especially HeII, CIII, NIII, OIII SiV, HeI, but HI is weak.

B Blue-white stars with surface temperatures of about 15,000 K. HeII lines have disappeared. HeI is strongest at B2. HI lines getting stronger. OII, Si II, and Mg II lins are visible.

A White stars with surface temperatures of about 9,000 K. The HI lines dominate the spectrum and are strongest at A0. HeI no longer visible. Neutral metal lines begin to appear.

F Yellow-white stars with surface temperatures of about 7,000 K. The HI lines are getting weaker while CaII lines are getting stronger. Many other metal lines such as FeI, FeII, CrII, and TiII are getting stronger.

Class and Temperature

G Yellow stars like the sun with surface temperatures of about 5,500 K. HI lines still getting weaker while CaII lines are strongest at G0. Metal lines are getting stronger.

K Yellow-orange stars with surface temperatures of about 4,000 K. Spectrum is dominated by metal lines. CaI getting stronger. TiO bands become visible at K5.

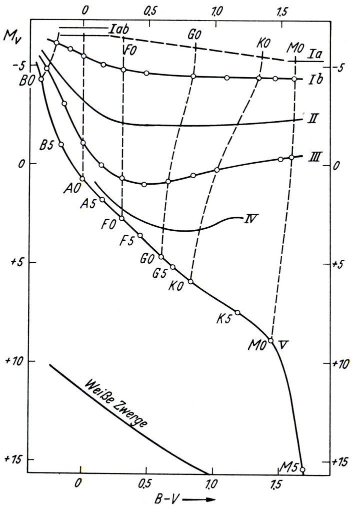
M Red stars with surface temperatures of about 3,000 K. TiO bands are very prominent. CaI at 423 nm is very strong. Many neutral metal lines are seen. For stars cooler than M4 absorption by TiO is so severe that it's very difficult to find the continuum.

Classification of Stars by Luminosity

Classification of stars by luminosity classes:

Ia –	Hypergiants
I	Supergiants
II	Bright Giants
III	Giants
IV	Subgiants
V	Main Sequence (Dwarfs)
VI	Subdwarfs.

Classification in the HRD



Spectral type and luminosity class in the HRD.

Planck Function - Limiting Cases

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\,, \quad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}\,,$$

Rayleigh-Jeans for long wavelength ($h\nu/kT \ll 1$):

$$B_\nu(T) = \frac{2h\nu^2 kT}{c^2}$$

Wien Limit for short wavelength ($h\nu/kT \gg 1$):

$$B_\nu(T) = \frac{2h\nu^3}{c^2}e^{-h\nu/kT}$$

Wien Displacement law:

$$\lambda_{\max} T = 2.9\times10^{-3}\mathrm{m\,K}\,, \quad \frac{c}{\nu_{\max}}T = 5.1\times10^{-3}\mathrm{m\,K}$$

Planck Spectrum