TIME SCALES FOR ENERGY

- GRAVITATIONAL E

(Approx.)

$$E_G = \frac{GM^2}{2R} \qquad G = 6.67 \times 10^{-8} \frac{cm^3}{g\, s^2}$$

How long does it take to radiate away this E?

$$\tau = E/L$$

IN CASE OF E_{GRAV}:

KELVIN-HELMHOLTZ TIME SCALE

$$\boxed{\tau_{KH} = \frac{E_G}{L} = \frac{GM^2}{2RL}}$$

time it takes star to radiate away gravit. E

→ also time-scale to get into gravo-thermal EQUILIBRIUM

or to evolve by contraction!

ESTIMATE FOR SUN

$$E_G \sim 1.89 \times 10^{48} \text{ erg}$$

$$\tau_{KH} = E_G / L_\odot = 4.9 \times 10^{14} \text{ s}$$

$$= 15.6 \times 10^{6} \text{ yr}$$

TRICK

$$\boxed{1 \text{ yr} \simeq \pi \times 10^{7} \text{ s}}$$

Basic Assumptions

- Stars evolve in isolation
  $\Leftrightarrow$ distance between stars >> Radii

- spherical symmetry
  Sun rotates once in 27 days
  $\rightarrow$ angular frequency $\omega = 2.5 \times 10^{-6}/s$

$\rightarrow$ compare Energy

$$\frac{E_{rot}}{E_{pot}} \simeq \frac{M\omega^2 R^2}{GM^2/R} \simeq 2 \times 10^{-5}$$

- small $\vec{B}$
  Even for $B \sim 0.1\ T$

$$\frac{E_{mag}}{E_{grav}} \simeq \frac{B^2 R^3/\mu_0}{GM^2/R} \sim 10^{-11}$$

- Neglect electric Field
  (though ionized gas)

STELLAR STRUCTURE

ASSUME <u>L</u>ocal <u>T</u>hermodynamic <u>E</u>quilibrium

→ gas & RADIATION IN EQUILIBRIUM

→ SAME, <u>UNIQUE</u> T

gas constituents in thermal equilibrium

- Different ion species have same T
- e^- have same T
- Ionization stages in equilibrium

[SAHA EQN.:

IONIZATION $\rightleftarrows$ e^- capture

→ mostly outer layers of stars

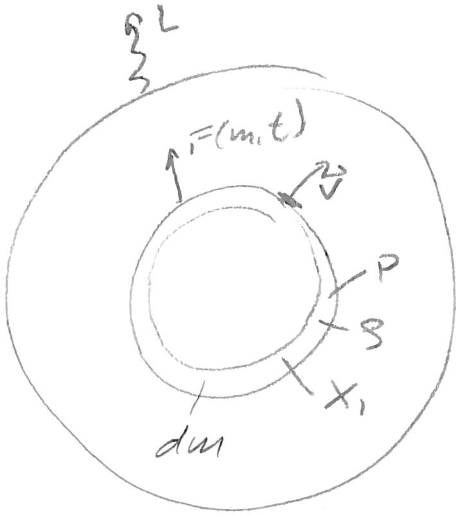
+ interior mostly fully ionized

Q: NAMING SCHEME FOR IONIZATION STAGES

Fe <u>IV</u>

MASS COORDINATE

SPHERICAL STARS



Thermodynamic T, S

For given composition X_i

$$\sum_{i=1}^{N} X_i = 1$$

$\boxed{N \text{ species}}$

+ DYNAMICS v_r

Q: How many quantities do we need to describe spherically symmetric star?

$$dm = 4\pi r^2 \rho(r)\, dr$$

$$\Leftrightarrow \quad m(r) = \int_0^r 4\pi r^2 \rho(r)\, dr$$

total mass $M = \int_0^R 4\pi r^2 \rho(r)\, dr$

TRANSFORMATION

$$\frac{\partial}{\partial r} = \frac{dm}{dr} \cdot \frac{\partial}{\partial m}$$

$$! \quad = 4\pi r^2 \rho \frac{\partial}{\partial m}$$

"Lagrange" COORDINATE

Q: how do we call coordinate system fixed in space?

! follows (traces) mass elements explicitly:
thermodynamics, energy, velocity, ion. stages, comp.
→ nuclear reactions

→ exchange "transport" modelled explicitly: Diffusion, convection

ENERGY EQUATION

1ST LAW OF Thermodynamics:

$$dU = dQ + dW$$

WORK DONE ON GAS

heat input into gas

use specific values

$$du = dq + dw$$

$$dW = -P\,dV$$

"specific volume" $v = 1/\rho$

$$dq = dt \cdot \left(\varepsilon - \frac{\partial L}{\partial m} \right)$$

[erg/g/s]

$L = L(m, t)$

also L_r, ℓ

"nuclear" energy generation rate

W/R TIME

$L(m,t) = L(t)$

$$\frac{\partial u}{\partial t} = \varepsilon_{nuc} - P \frac{\partial v}{\partial t} - \frac{\partial L}{\partial m}$$

$$\Leftrightarrow \quad \frac{\partial L}{\partial m} = -P \frac{\partial v}{\partial t} + \varepsilon - \frac{\partial u}{\partial t}$$

Later: $\varepsilon = \varepsilon_{nuc} - \varepsilon_{\nu}$

in thermal equilibrium:

$$\frac{\partial}{\partial t} \longmapsto 0$$

$$\rightarrow \quad \varepsilon = \frac{dL}{dm}$$

$$\rightarrow L(M) = L = \int_0^M \varepsilon \, dM$$

if powered by nuclear burning:

$$L_{nuc} = \int_0^M \varepsilon_{nuc} \, dm = L(M)$$

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average ε of sun: $L_\odot / M_\odot \sim 2$ erg/g

$$\varepsilon = \varepsilon_{nuc} - \varepsilon_\nu$$

Sun: $\frac{\varepsilon_\nu}{\varepsilon} \sim 7\%$!

- What needs to be in "$\mathcal{E}$"

→ terms that change on time-scale on which star evolves

IN OUR CASE: nuclear reactions

- things that change much slower than star evolves: we do not need to include (obviously)
- things that change much faster: we assume to be "in equilibrium"

EXAMPLES:
- time for gas to set into LTE → EOS
- IONIZATION EQUILIBRIUM → EOS
- NUCLEAR EXCITED STATES → IMPLICIT IN REACTION RATES [FOR THE MOST PART]
- IN <u>N</u>UCLEAR <u>S</u>TATISTICAL <u>E</u>QUILIBRIUM

→ ONLY NEED $\underbrace{\#n/\#p}_{Y_e}$ → EOS

$\frac{\partial}{\partial t}$

MOMENTUM EQUATION / EQUATION OF MOTION

$$\frac{\partial^2}{\partial t^2} r = -\frac{Gm}{r^2} - \frac{1}{\rho}\frac{\partial P}{\partial r} \qquad \Big| \; ! \; \frac{dP}{dr} < 0$$

USE $\frac{dm}{dr} = 4\pi r^2 \rho \longrightarrow \frac{\partial}{\partial r} = 4\pi r^2 \rho \frac{\partial}{\partial m}$

$$\ddot{r} = -\frac{Gm}{r^2} - 4\pi r^2 \frac{\partial P}{\partial m}$$

$$\longrightarrow \frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

$$= -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial v}{\partial t}$$

$v = \dot{r}$

$\boxed{!} = \left.\frac{\partial r}{\partial t}\right|_m$

$$\frac{\partial r}{\partial t} = v$$

HYDROSTATIC EQUILIBRIUM

hydrostatic: $\ddot{r} = 0$

$$\rightarrow \frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \qquad \Big| \qquad \frac{dP}{dr} = -\rho\frac{Gm}{r^2} = g\rho$$

$g = -\frac{Gm}{r^2}$

$\boxed{\text{DEF}}$

$\rightarrow$ PRESSURE SCALEHEIGHT (in HSE)

$$H_p = -\frac{dr}{d\ln P} = -P \cdot \frac{dr}{dP} = \frac{P}{g\rho}$$

MATH NOTE

$$d \ln x = \frac{1}{x}\, dx$$

$$\frac{d \ln x}{d \ln y} = \frac{y}{x} \frac{dx}{dy}$$