$\boxed{\text{RECAP}}$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

$$= -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial v}{\partial t}$$

$$v = \left.\frac{\partial r}{\partial t}\right|_m$$

confusing...

$$\frac{\partial L}{\partial m} = \varepsilon - \frac{\partial u}{\partial t} - P\frac{\partial V}{\partial t} \qquad \Big| \; V = 1/\rho$$

$$= \varepsilon \underbrace{- \frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial}{\partial t}\rho}_{dq/dt} \qquad \Big| \; d\frac{1}{\rho} = -\frac{1}{\rho^2}d\rho$$

NOTE : $H_p = -\frac{dr}{d\ln P} \overset{\text{Hydrostatic}}{=} \frac{P}{\rho g}$

$$\boxed{\frac{\partial L}{\partial m} = \varepsilon - T\frac{ds}{dt}}$$

ESTIMATE OF CENTRAL PRESSURE

$$P(M) - P(0) = -\int_0^M \frac{Gm\,dm}{4\pi r^4} \quad \Big| \; r < R$$
$\simeq 0$

$$P(m=0) > \int_0^M \frac{Gm\,dm}{4\pi R^4} = \frac{GM^2}{8\pi R^4}$$

$$= 4.4 \times 10^{14} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R_\odot}{R}\right)^4 \frac{dyn}{cm^2}$$

Q: PRESSURE GRADIENT @ CENTER?

$$\frac{\partial P}{\partial r} = - \rho \frac{Gm}{r^2} \quad \Big| \; \rho \sim \rho_c$$

$$m = \frac{4\pi}{3} \rho r^3$$

$$\sim \frac{4\pi}{3} \rho^2 r \quad \rightarrow \quad \frac{dP}{dr} = 0$$

$$\frac{\partial P}{\partial m} = - \frac{Gm}{4\pi r^4}, \quad r = \sqrt[3]{\frac{3m}{4\pi\rho}} \sim C \cdot m^{1/3}$$

$\rho(m)$?

THERMODYNAMIC RELATION KW §4

$$\alpha = \left.\frac{\partial \ln \rho}{\partial \ln P}\right)_T$$

$$\boxed{\begin{aligned} dv &= -\frac{1}{\rho^2} d\rho \\ \frac{\partial}{\partial v} &= -\rho^2 d\rho \end{aligned}}$$

$$\delta = -\left.\frac{\partial \ln \rho}{\partial \ln T}\right)_P$$

EOS: $$\frac{d\rho}{\rho} = \alpha \frac{dP}{P} - \delta \cdot \frac{dT}{T}$$

$$d\ln \rho = \alpha \, d\ln P - \delta \, d\ln T$$

SPECIFIC HEATS:

$$-\frac{P}{\rho^2}\left.\frac{\partial \rho}{\partial T}\right)_P$$

$$c_p = \left.\frac{dq}{dT}\right)_T = \left.\frac{\partial u}{\partial T}\right)_P + P\left.\frac{\partial v}{\partial T}\right)_P$$

$$c_v = \left.\frac{dq}{dT}\right)_{v \to \rho} = \left.\frac{\partial u}{\partial T}\right)_v$$

$$du = \left.\frac{\partial u}{\partial v}\right)_T dv + \left.\frac{\partial u}{\partial T}\right)_v dT$$

$$= \left.\frac{\partial u}{\partial \rho}\right)_T d\rho + \left.\frac{\partial u}{\partial T}\right)_\rho dT \qquad ?$$

DERIVE FORMULATION OF L EQN:

$$ds = \frac{dq}{T} \leftarrow \quad dq = T\,ds \leftarrow \text{SPECIFIC ENTROPY}$$

$$= \frac{1}{T}(du + P\,dv) \leftarrow \quad = du + P\,dv$$

$$= \frac{1}{T}\left[\frac{\partial u}{\partial v}\Big)_T + P\right] dv + \frac{1}{T}\frac{\partial u}{\partial T}\Big|_V dT$$

NOTE $$\frac{\partial^2 s}{\partial T \partial v} = \frac{\partial^2 s}{\partial v \partial T}$$

$$\frac{\partial}{\partial T}\left[\frac{1}{T}\frac{\partial u}{\partial v}\Big)_T + \frac{P}{T}\right] = \frac{1}{T}\frac{\partial^2 u}{\partial T \partial v}$$

$$-\frac{1}{T^2}\frac{\partial u}{\partial v}\Big)_T + \frac{1}{T}\frac{\partial^2 u}{\partial T \partial v} - \frac{P}{T^2} + \frac{1}{T}\frac{\partial P}{\partial T}\Big|_T = \frac{1}{T^2}\frac{\partial^2 u}{\partial T \partial v}$$

$| \times -T^2$

$$\frac{\partial u}{\partial v}\Big)_T = T\frac{\partial P}{\partial T}\Big|_V - P$$

FROM $$du = \frac{\partial u}{\partial T}\Big|_V dT + \frac{\partial u}{\partial V}\Big)_T dv$$ | ASSUME P, T AS VARIABLES

$$\rightarrow \frac{du}{dT} = \frac{\partial u}{\partial T}\Big|_V + \frac{\partial u}{\partial v}\Big)_T \frac{dv}{dT}$$ REPLACE

$$\rightarrow \frac{\partial u}{\partial T}\Big|_P = \frac{\partial u}{\partial T}\Big|_V + \frac{\partial u}{\partial v}\Big)_T \frac{\partial v}{\partial T}\Big|_P$$

$$= \frac{\partial u}{\partial T}\Big)_V + \frac{\partial v}{\partial T}\Big)_P\left[T\frac{\partial P}{\partial T}\Big)_V - P\right]$$

NOW

$$C_p - C_v = \frac{\partial u}{\partial T}\Big)_P + P\frac{\partial v}{\partial T}\Big)_P - \frac{\partial u}{\partial T}\Big)_v$$

$$= \frac{\partial v}{\partial T}\Big)_P \frac{\partial P}{\partial T}\Big)_v T$$

?

AND

$$\frac{\partial P}{\partial T}\Big)_v = -\frac{\frac{\partial v}{\partial T}\Big)_P}{\frac{\partial v}{\partial P}\Big)_T} = \frac{P\delta}{T\alpha}$$

$$\frac{\partial P}{\partial T}\Big)_V = \frac{\partial(P,V)}{\partial(T,V)} = \frac{\partial(V,P)}{\partial(T,P)} \cdot \frac{\partial(T,P)}{\partial(V,T)} = -\frac{\partial(V,P)}{\partial(T,P)} \cdot \frac{\partial(P,T)}{\partial(V,T)}$$

$$= -\frac{\partial(V,P)}{\partial(T,P)}\left[\frac{\partial(V,T)}{\partial(P,T)}\right]^{-1} = -\frac{\partial v}{\partial T}\Big)_P \Big/ \frac{\partial v}{\partial P}\Big)_T$$

$$C_P - C_V = T\frac{\partial v}{\partial T}\Big)_P \frac{P\delta}{T\alpha}$$

$$\Big|\ T\frac{\partial v}{\partial T}\Big)_P = v\cdot\delta = \delta/\rho$$

$$= \frac{P\delta^2}{\rho T\alpha}$$

IDEAL GAS: $\quad C_p - C_v = \mathcal{R}/\mu$

$$dq = u + P\,dv = \frac{\partial u}{\partial T}\Big)_V dT + \left[\left(\frac{\partial u}{\partial V}\right)_T + P\right] dV$$

$$= \frac{\partial u}{\partial T}\Big)_V dT + T\,\frac{\partial P}{\partial T}\Big)_V dv$$

$$= C_V\,dT - \frac{T}{\rho}\left(\frac{\partial P}{\partial T}\right)_V \frac{d\rho}{\rho} = C_V\,dT - \frac{P\delta}{\rho\alpha}\,\frac{d\rho}{\rho}$$

$$= C_V\,dT - \frac{P\delta}{\rho\alpha}\left(\alpha\,\frac{dP}{P} - \delta\,\frac{dT}{T}\right)$$

$$= \left(C_V + \frac{P\delta^2}{\rho T\alpha}\right) dT - \frac{\delta}{\rho}\,dP$$

$$= C_P\,dT - \frac{\delta}{\rho}\,dP$$

$$\rightarrow \quad \frac{\partial L}{\partial m} = \varepsilon - T\,\frac{ds}{dt} = \varepsilon - \frac{dq}{dt}$$

$$\boxed{\frac{\partial L}{\partial m} = \varepsilon - C_P\,\frac{dT}{dt} + \frac{\delta}{\rho}\,\frac{\partial P}{\partial t}}$$

NOTE <u>ADIABATIC T GRADIENT:</u>

$$ds = 0 \rightarrow dq = T\,ds = 0 = c_P\,dT - \frac{\delta}{\rho}\,dP$$

DEF: $$\nabla_{AD} := \frac{\partial \ln T}{\partial \ln P}\Big)_S = \frac{P}{T}\,\frac{dT}{dP}\Big)_S = \frac{P\delta}{T\rho C_P}$$

DEF:

GRAVITATIONAL E RELEASE:

$$\varepsilon_g = -T \frac{\partial s}{\partial t}$$

$$= -C_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho} \frac{\partial P}{\partial t}$$

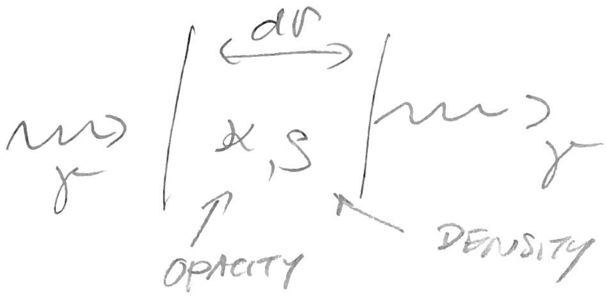
$$= -C_P T \left(\frac{1}{T} \frac{\partial T}{\partial t} - \frac{\nabla_{ad}}{P} \frac{\partial P}{\partial t} \right)$$

$ds = \frac{dq}{T}$

$$\frac{\partial L}{\partial m} = \varepsilon + \varepsilon_g$$

NOTE $\varepsilon = \varepsilon_{nuc} - \varepsilon_\nu$

Energy Transport



Flux $F = \dfrac{L_r}{4\pi r^2}$ $\qquad \dfrac{\text{energy}}{\text{time} \times \text{AREA}}$

$$dF = -\kappa \cdot \rho \cdot F \, dr$$

For const opacity: $F(r) = F_0 \cdot e^{-\underbrace{\kappa \cdot \rho}_{\text{characteristic absorption length}} \cdot (r - r_0)}$

$= F_0 \cdot e^{-\tau}$

Def: Optical Depth: $d\tau = -\kappa \cdot \rho \, dr$

Sources of opacity:

- electron scattering (Thomson scattering)
  rel: Compton scattering
  $\kappa_\nu = \kappa_{es,0}/\mu_e \simeq \dfrac{\kappa_{es,0}}{2}(1+X) \simeq 0.2 \, \dfrac{cm^2}{g}(1+X)$
- free-free absorption
  (interaction of free e− with atom/ion)
  Rev: Bremsstrahlung
- Bound-free absorption:
  (rev: recombination
- Bound-bound absorption
  (rev: de-excitation)

$\kappa_{es,0} = \dfrac{\sigma_T}{u}$ $\qquad u = \text{AMU} = \frac{1}{12}(m_{^{12}C})$

$\sigma_T = \dfrac{8\pi}{3}\left(\dfrac{e^2}{m_e c^2}\right)^2$

$\simeq 6.65 \times 10^{-25} \, cm^2$