DIFFUSION OF RADIATION

DIFFUSIVE FLUX: $j = -D \cdot \nabla u$

WITH DIFFUSION COEFFICIENT: $D = \frac{1}{3} v \cdot \ell$

NOW REPLACE

u by $U = aT^4$

v by c

ℓ by $\ell_{ph} = \frac{1}{\kappa \cdot \rho}$

RADIAL COMPONENT $F_r = |\vec{F}| =: F \leftarrow j$

$$\nabla U \rightarrow \frac{\partial U}{\partial r}$$

$$\frac{\partial u}{\partial r} = 4aT^3 \frac{\partial T}{\partial r}$$

$$\rightarrow F = -\frac{4ac}{3} \frac{T^3}{\kappa\rho} \frac{\partial T}{\partial r} \qquad \Big| \; dm = 4\pi r^2 \rho \, dr$$

$$= -\frac{16\pi a c r^2}{3\kappa} \frac{\partial T}{\partial m}$$

RECALL: CONDUCTION

$$\vec{F} = -K_{RAD} \nabla T$$

WE NOW DEFINE: $K_{RAD} = \frac{4ac}{3} \frac{T^3}{\kappa\rho}$

LOCAL LUMINOSITY $L = 4\pi r^2 F$

$$\frac{\partial T}{\partial r} = -\frac{3}{16\pi ac} \frac{\kappa\rho}{r^2} \frac{L}{T^3}$$

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac} \frac{\kappa}{r^4} \frac{L}{T^3}$$

$dm = 4\pi r^2 \rho \, dr$

$\frac{\partial}{\partial m} = \frac{1}{4\pi r^2\rho} \frac{\partial}{\partial r}$

$\frac{\partial}{\partial r} = 4\pi r^2 \rho \frac{\partial}{\partial m}$

IN HSE: $\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$

$$\frac{\partial T}{\partial m} = +\frac{3}{16\pi acG} \frac{\kappa L}{mT^3} \frac{\partial P}{\partial m}$$

DEF: T gradient due to radiation:

$$\nabla_{rad} := \left.\frac{d\ln T}{d\ln P}\right|_{RAD} = \frac{3}{16\pi acG} \frac{\kappa L P}{m T^4}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^2 P} \nabla_{RAD}$$

NOT AN OPERATOR

Q: HOW WOULD THIS LOOK WITH ACCELERATION?

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

$\rightarrow$ FIND $\frac{\partial T}{\partial m}$!

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} = -\frac{Gm}{4\pi r^4}\left(1+\frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right)$$

$$\rightarrow \frac{\partial T}{\partial m} = \frac{3}{16\pi a c G}\frac{\kappa L}{m T^3}\frac{\partial P}{\partial m}\left(1+\frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right)$$

DEF. $\nabla_{RAD} = \left.\frac{\partial \ln T}{\partial \ln P}\right)_{rad} = \frac{3}{16\pi a c G}\frac{\kappa L P}{T^4}\Big/\left(1+\frac{r^2 \partial^2 r}{Gm\,\partial t^2}\right)$

AND WE WRITE:

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla_{rad}\left[1+\frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right]$$

Conductive E Transport

So far: $F = F_{RAD}$

Same for conductive flux:

$$F_{CD} = - K_{CD} \cdot \nabla T$$

So, we add both:

$$F = F_{RAD} + F_{CD} = - (K_{RAD} + K_{CD}) \cdot \nabla T$$

We may write formally:

$$K_{CD} = \frac{4ac}{3} \frac{T^3}{\kappa_{CD}\, \rho}$$

$\rightarrow$ Total flux: $F = - \frac{4ac}{3} \frac{T^3}{\rho} \left(\frac{1}{\kappa_{RAD}} + \frac{1}{\kappa_{CD}} \right) \nabla T$

Def: $\frac{1}{\kappa} := \frac{1}{\kappa_{RAD}} + \frac{1}{\kappa_{CD}}$ (Resistors in parallel...)

$$\rightarrow F = - \frac{4ac}{3} \frac{T^3}{\kappa \rho} \nabla T$$

ROSSELAND MEAN OPACITY

[KW §5]

IN REALITY: all quantities are a function of photon frequency, ν:
l_{ph}, H_ν, D_ν, U_ν, κ_ν

$\rightarrow F_\nu = -D_\nu \nabla U_\nu$

$\rightarrow D_\nu = \frac{1}{3} c \cdot l_{\nu,ph} = \frac{c}{3\kappa_\nu \rho}$

$$U_\nu = \underset{\substack{\uparrow \\ \text{WHY?}}}{\frac{4\pi}{c}} B(\nu,T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/k_BT} - 1}$$

↖ PLANCK FUNCTION FOR INTENSITY

$\rightarrow \nabla U_\nu = \frac{4\pi}{c} \frac{\partial B}{\partial T} \nabla T \rightarrow F_\nu = -\left(\frac{1}{\kappa_\nu} \frac{\partial B}{\partial T}\right) \frac{4\pi}{3\rho} \nabla T$

INTEGRATE OVER <u>ALL</u> FREQUENCIES:

$$F = -\underbrace{\left[\frac{4\pi}{3\rho} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B}{\partial T} d\nu\right]}_{=: K_{RAD}} \nabla T \overset{!}{=} -K_{RAD} \nabla T$$

$$K_{RAD} = \frac{4\pi}{3\rho} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu}{\partial T} d\nu \overset{!}{=} \frac{4ac}{3} \frac{T^3}{\kappa \rho}$$

$\rightarrow$ DEF: $\frac{1}{\kappa} = \frac{\pi}{acT^3} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B}{\partial T} d\nu$

<u>ROSSELAND MEAN OPACITY</u>

! FREQUENCIES OF MAX FLUX ARE FAVORED IN MEAN!

NOTE:

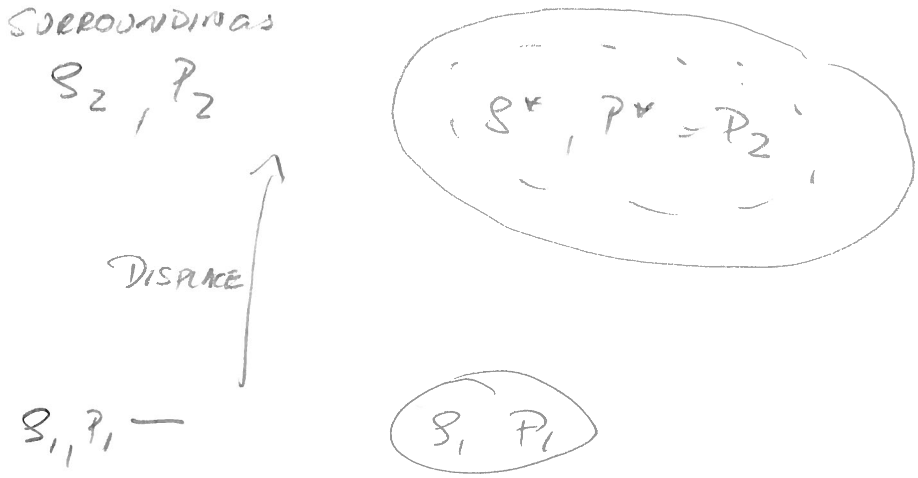
$$\frac{acT^3}{4} = \int_0^\infty \frac{\partial B}{\partial T}\, d\nu$$

$$\rightarrow \quad \frac{1}{\kappa} = \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B}{\partial T}\, d\nu \Big/ \int_0^\infty \frac{\partial B}{\partial T}\, d\nu$$

is harmonic mean

Convection

Rising Bubble Model



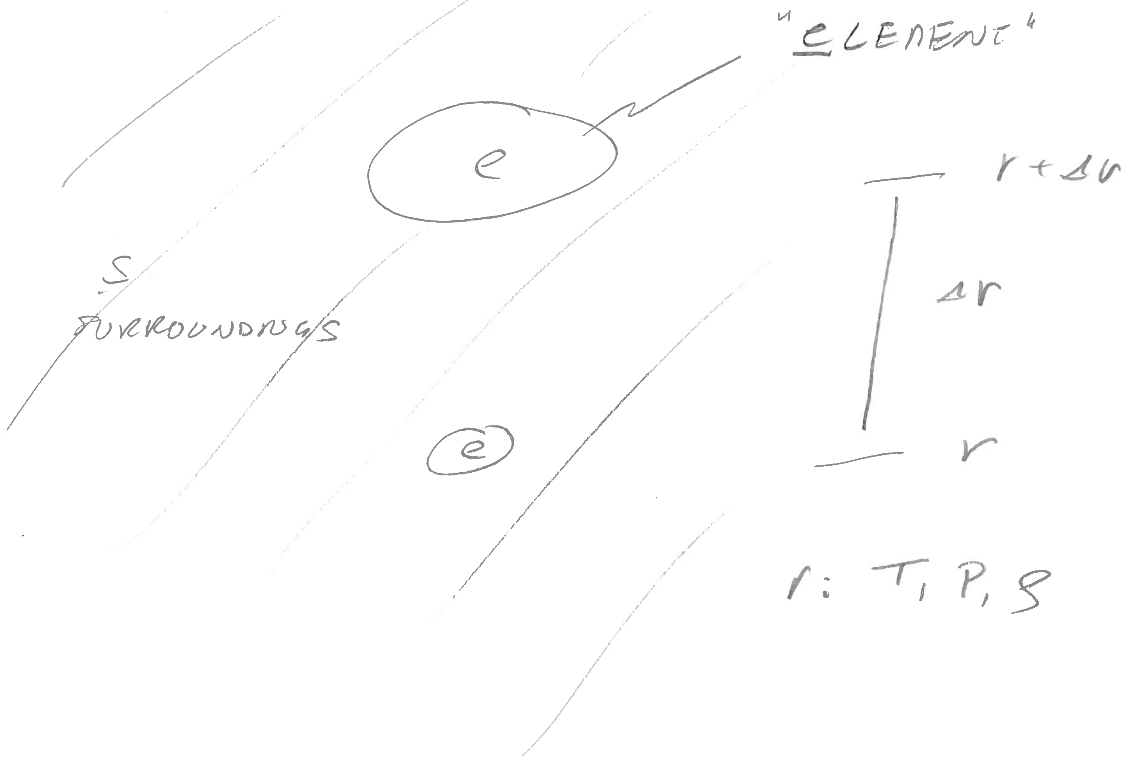
$\uparrow \rho_* < \rho_2$

$\downarrow \rho_* > \rho_2$

Assume:
- P equilibrium
- No heat exchange
  - $\Leftrightarrow$ Adiabatic expansion
  - $\Leftrightarrow$ $S =$ const $\leftarrow$ What could go wrong?
- No composition exchange

$\rightarrow$ New $\rho <$ surroundings : cont to rise

$\rho >$ surroundings : drop back



COMPARE ELEMENT (BUBBLE) at LOCATIONS
r and $r+\Delta r$

$$\Delta\rho = \left[\left.\frac{d\rho}{dr}\right)_e - \left.\frac{d\rho}{dr}\right)_s \right] \Delta r$$

$\rightarrow$ STABILITY IF $\Delta\rho > 0$ (for all Δr)

$$\leftrightarrow \quad \left.\frac{d\rho}{dr}\right)_e - \left.\frac{d\rho}{dr}\right)_s > 0$$

for
<u>STABILITY</u>

NOW. USE EQUATION OF STATE FOR $\frac{d\rho}{dr}$

RECALL:
$$\frac{d\rho}{\rho} = \alpha \frac{dP}{P} - \delta \cdot \frac{dT}{T} + \varphi \cdot \frac{d\mu}{\mu}$$

WITH $\alpha = \left.\frac{\partial \ln \rho}{\partial \ln P}\right|_{T,\mu}$, $\delta = \left.\frac{d \ln \rho}{d \ln T}\right|_{P,\mu}$

$\varphi = \left.\frac{\partial \ln \rho}{\partial \ln \mu}\right|_{P,T}$, $\mu = 1 / \sum X_i \frac{Z_i + 1}{A_i}$

IDEAL GAS $P = \frac{R T \rho}{\mu}$

ASSUME COMP. DOES NOT CHANGE IN ELEMENT

$$0 < \frac{\alpha}{P} \left.\frac{dP}{dr}\right|_e - \frac{\delta}{T} \left.\frac{dT}{dr}\right|_e - \frac{\alpha}{P} \left.\frac{dP}{dr}\right|_s + \frac{\delta}{T} \left.\frac{dT}{dr}\right|_s - \frac{\varphi}{\mu} \left.\frac{d\mu}{dr}\right|_s$$

$$\frac{\varphi}{\mu} \left.\frac{d\mu}{dr}\right|_e = 0$$

FOR PRESSURE EQ: $\frac{\alpha}{P} \left(\frac{dP}{dr}\right)_e = \frac{\alpha}{P} \left.\frac{dP}{dr}\right|_s$ cancel out

$$-\frac{\delta}{T} \left.\frac{dT}{dr}\right|_e + \frac{\delta}{T} \left.\frac{dT}{dr}\right|_s - \frac{\varphi}{\mu} \left.\frac{d\mu}{dr}\right|_s > 0$$

SIMPLIFY: multiply by $H_P = -\frac{dr}{d\ln P} = -P\frac{dr}{dP} > 0$

NOTE $H_P = \frac{P}{g\rho}$

$$\left.\frac{d\ln T}{d\ln P}\right)_s < \left.\frac{d\ln T}{d\ln P}\right)_e + \frac{\varphi}{\delta}\left.\frac{d\ln\mu}{d\ln P}\right)_s$$

INTRODUCE

$$\nabla = \left.\frac{d\ln T}{d\ln P}\right)_s, \quad \nabla_e = \left.\frac{d\ln T}{d\ln P}\right)_e$$

$$\nabla_\mu = \left.\frac{d\ln\mu}{d\ln P}\right)_s$$

$$\rightarrow \boxed{\nabla < \nabla_e + \frac{\varphi}{\delta}\nabla_\mu}$$

ASSUME E move adiabatically, SURROUNDING IS RADIATIVE, USE

$$\nabla_{ad} = \left.\frac{\partial\ln T}{\partial\ln P}\right)_{ad} \qquad \nabla_{RAD} = \left.\frac{d\ln T}{d\ln P}\right)_{RAD,\,STAR}$$

PROPERTY OF GAS

$\rightarrow \nabla_e \Rightarrow \nabla_{ad}, \quad \nabla \rightarrow \nabla_{RAD}$

$$\boxed{\nabla_{RAD} < \nabla_{AD} + \frac{\varphi}{\delta}\nabla_\mu}$$

LEDOUX CRITERION FOR STABILITY!