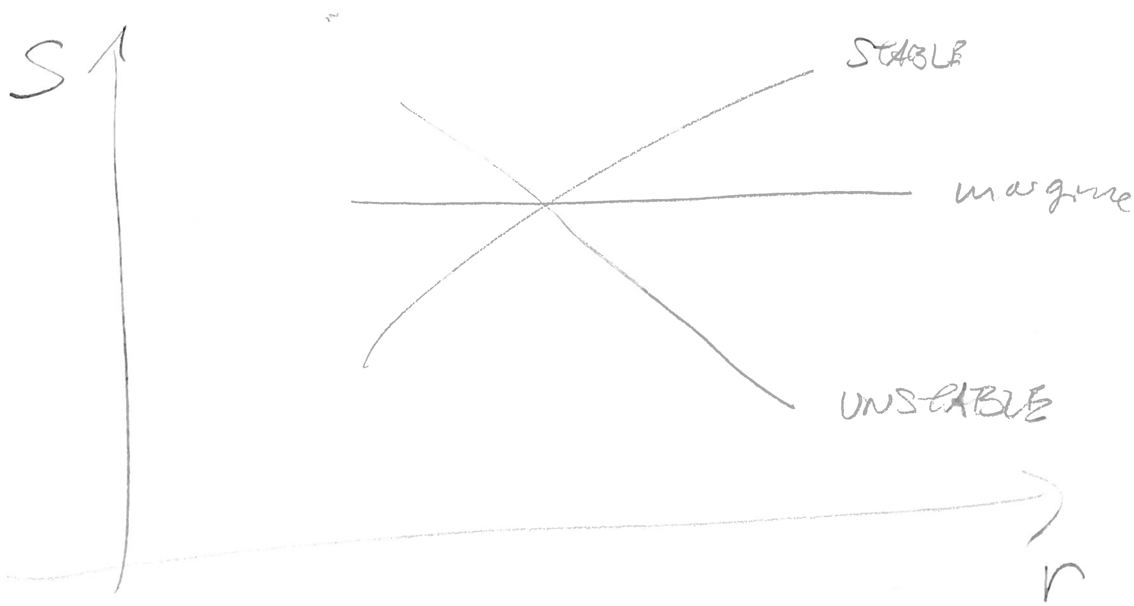
SUPERADIABATIC $\nabla > \nabla_{ad}$

$\rightarrow$ UNSTABLE

SUB-ADIABATIC $\nabla < \nabla_{ad}$

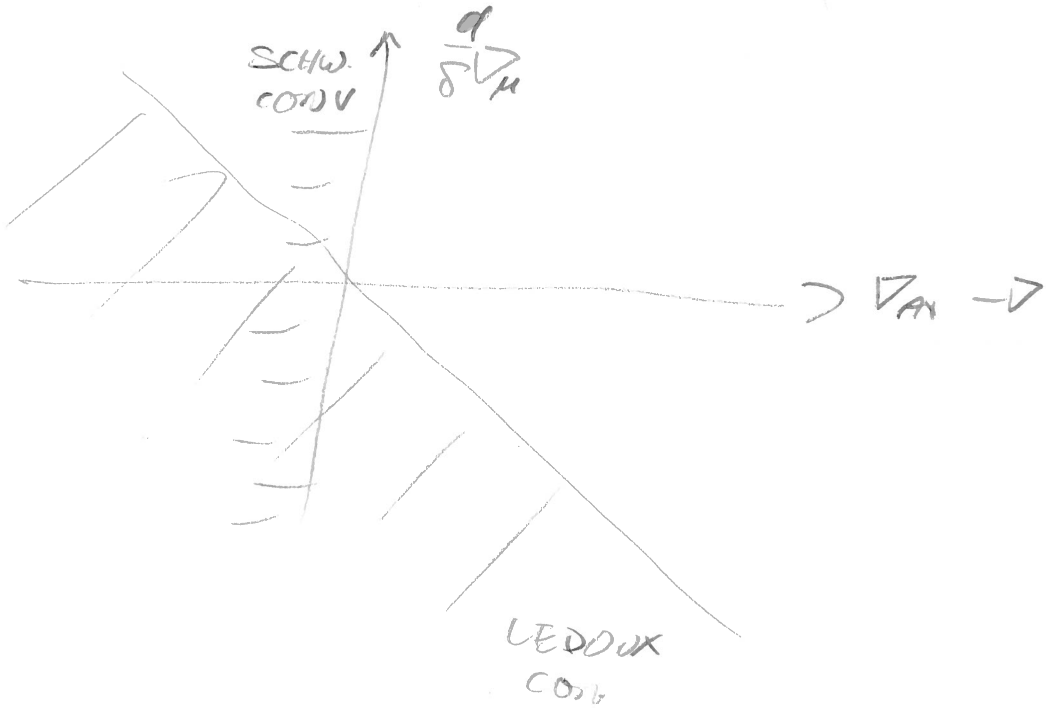
$\rightarrow$ STABLE



SCHWARZSCHILD CRITERION:

IF NO COMPOSITION GRADIENT IS PRESENT

$$\boxed{\nabla_{rad} < \nabla_{ad}}$$



NOTE. SCHW. CONV. IS APPROX FOR NO COMP. GRAD. THERE IS ONLY ONE CRITERION. LEDOUX!

MORE LOCAL INST.

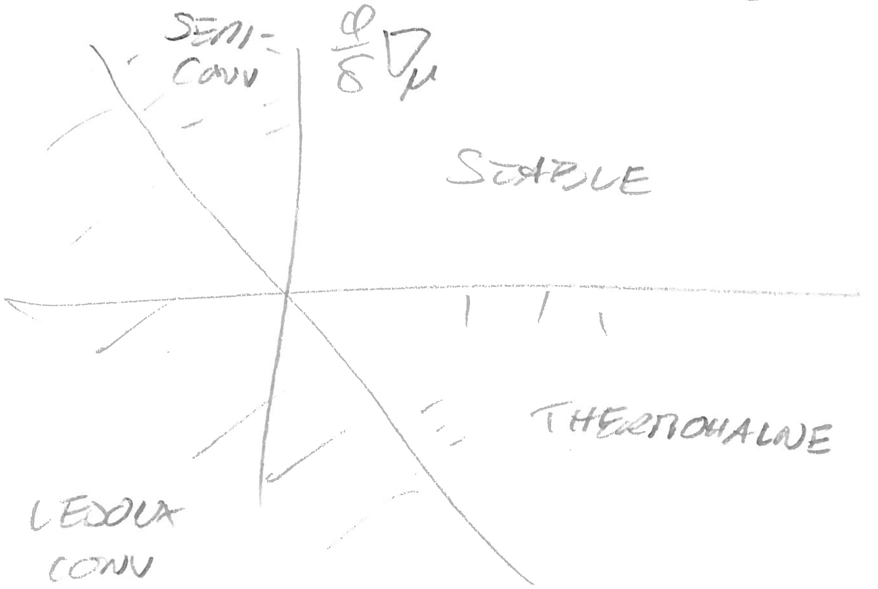
- SEMICONVECTION: WHERE

$$\frac{\varphi}{\delta}\nabla_\mu > 0 \,, \quad \nabla_{RAD} < \nabla_{AD} + \frac{\varphi}{\delta}\nabla_\mu$$

- THERMOHALINE CONV (SALT FINGER INST.)

WHERE

$$\frac{\varphi}{\delta}\nabla_\mu < 0 \,, \quad \nabla_{RAD} < \nabla_{AD} + \frac{\varphi}{\delta}\nabla_\mu$$



→ RESULT OF CONVECTION:
TYPICALLY CLOSE TO ADIABATIC STRATIFICATION, CHEMICALLY HOMOGENEOUS

→ MIXING LENGTH THEORY

KW §7

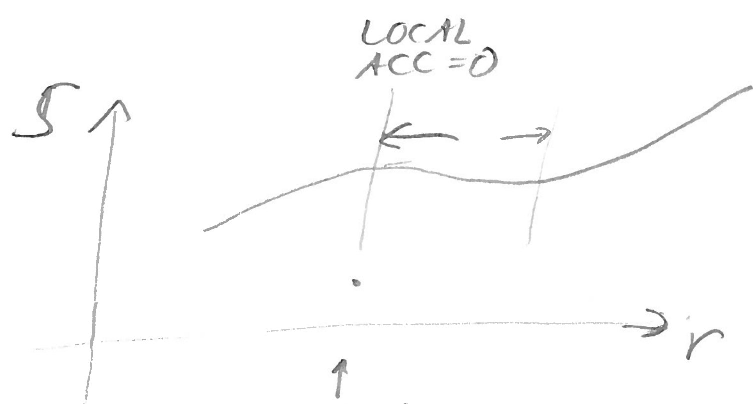
NOTES

SC : LAYERFORMATION ↗ LAYER MERGING → CONVECTION

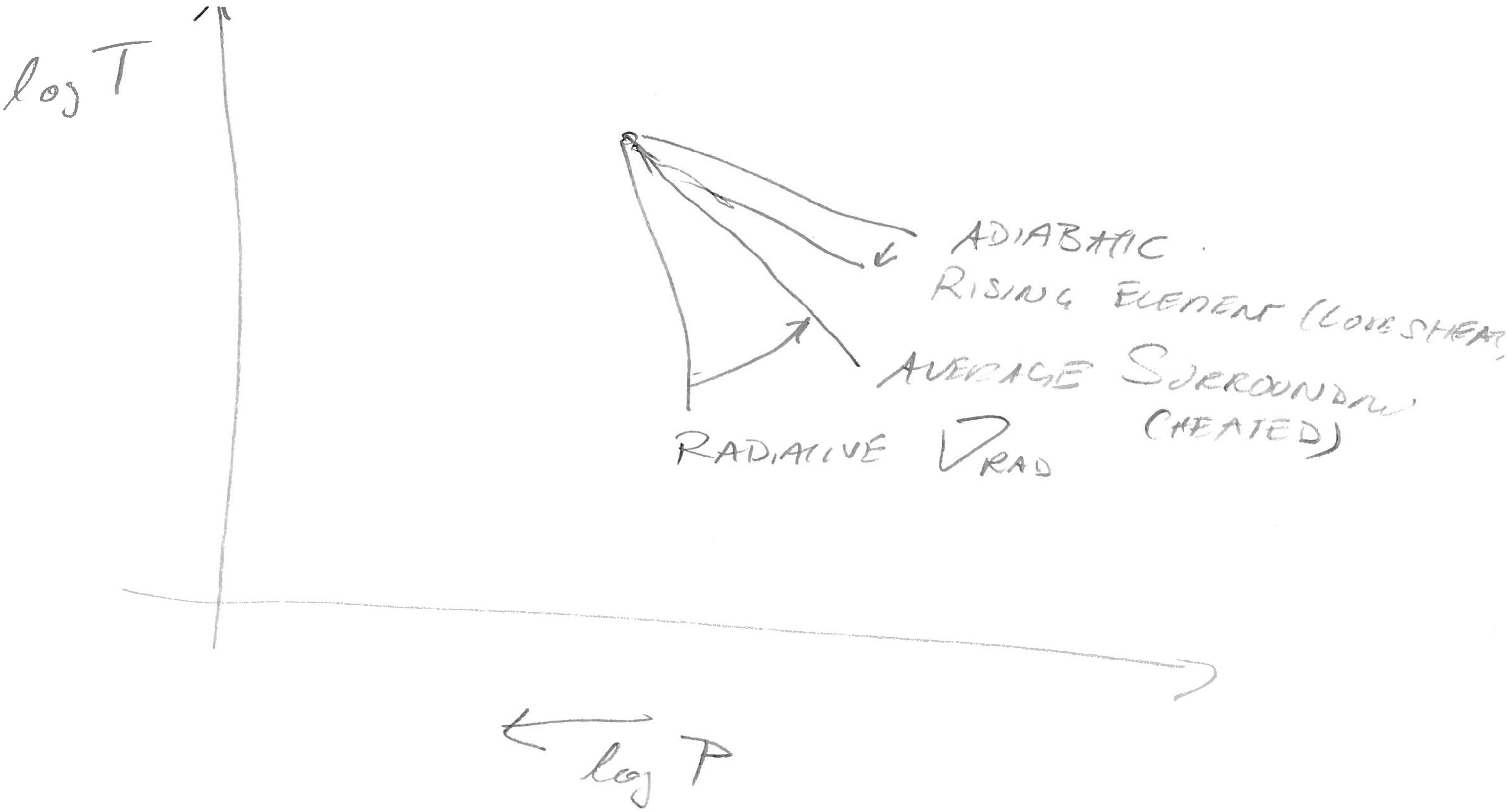
OVERSTABILITY ↑

CONVECTIVE PENETRATION

CONVECTIVE OVERSCHOOTING



↑ V IS NOT → INERTIA
→ OVERSHOOTING



NOTE: USUALLY IN CONVECTIVE REGIONS:

VERY SMALL DEVIATION FROM ∇_{ad}

→ CONVECTION IS EFFICIENT

WHERE NOT: "SUPERADIABATIC CONVECTION"

E.G.: SURFACE CONVECTION ZONES

→ SEE MLT (KW § 7)

COMPOSITION CHANGE

$$\frac{\partial}{\partial t} X_i = f_i(\rho, T, \vec{X})$$

COMPOSITION VECTOR X_i

$$= f_{i,nuc}(\rho, T, \vec{X}) + f_{i,mix}(\rho, T, \vec{X})$$

OFTEN: APPROXIMATE BY DECOUPLING

$$\frac{\partial}{\partial t} X_i = f_{i,nuc}(\rho, T, \vec{X}) - \frac{\partial}{\partial m}\left(D_m \frac{\partial}{\partial m} X_i\right)$$

FROM PHYSICAL
PROCESSES
LIKE CONVECTION

SUMMARY: STELLAR STRUCTURE EQUATIONS

STATIONARY STRUCTURE / HYDROSTAT	TIME-DEP. TERMS
$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$	
$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$	$- \frac{1}{4\pi r^2} \frac{\partial^2 r}{\partial t^2}$
$\frac{\partial L}{\partial m} = \varepsilon_{nuc} - \varepsilon_\nu$	$- c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho} \frac{\partial P}{\partial t}$
$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P} \nabla \times$	$\left[1 + \frac{r^2 \partial^2 r}{Gm\, \partial t^2} \right]$
$\frac{\partial X_i}{\partial t} =$	$f_i(\rho, T, \vec{X})$

EVOLUTION OR THERMAL time SCALE

$$\nabla = \begin{cases} \nabla_{RAD} = \left.\frac{d\ln T}{d\ln P}\right|_{RAD} = \frac{3}{16\pi a c G} \frac{\kappa L P}{m T^4} \\ \nabla_{ad} = \left.\frac{\partial \ln T}{\partial \ln P}\right|_{ad} \end{cases}$$

GOOD APPROX IN EFFICIENT CONVECTION ZONES

Q: what happens in inefficient convection zones

USUALLY: $\min\{\nabla_{ad}, \nabla_{RAD}\} \le \nabla \le \max\{\nabla_{ad}, \nabla_{RAD}\}$

A SIMPLE STELLAR MODEL (KW § 19)

ASSUMPTIONS:

- T, ϱ, P decrease outward
- L increases outward
- spherical symmetry

→ try first stationary time-independent SOLUTIONS

BOUNDARY CONDITIONS: (needed to solve PDE system)

we have

- center: $r = 0 = m = L$
- surface: $r = R$, $m = M$, $L = L_*$

effective T: $L = 4\pi R_{(eff)}^2 \sigma T_{eff}^4$

(will not need that here at first)

EOS: (in general) – will discuss later

$$P = \frac{\mathcal{R}}{\mu_I} \varrho T + P_e + \frac{1}{3} a T^4$$

IONS RADIATION

P_e: · ideal electron gas
· non-rel. deg. e^- gas
· rel.-deg e^- gas
· rel non-deg e^- gas (e^+e^- pairs) high T

SIMPLIFICATIONS: NOTE: EQN 1, 2, 3 only couple indirectly through dep. of $P(\rho, T)$

ASSUME P only depends on ρ
(or P can be expressed as fn of ρ)
$\rightarrow$ T-dep IMPLICIT !

!

ASSUME Power Law: $\boxed{P = K \cdot \rho^{\gamma}}$

NOTE: This is not an adiabatic index
$\Leftrightarrow$ we can have changes of entropy S

IT IS CALLED POLYTROPIC EXPONENT
EOS

(stellar models do take into account ∇S)

POLYTROPIC INDEX $\gamma = 1 + \frac{1}{n} = \frac{n+1}{n}$

SIMPLE SOLUTIONS

STARTING WITH HYDROSTATIC EQN. (radius formulation)

$$\frac{\partial P}{\partial r} = -\rho \frac{Gm}{r^2} \quad \Big| \times \frac{r^2}{\rho} \quad \Big| \frac{d}{dr}$$

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -G\frac{dm}{dr} \quad \Big| \; dm = 4\pi r^2 \rho \, dr$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho \quad \Big| \; \text{USE } P = K\rho^{\gamma} = K\rho^{\frac{n+1}{n}}$$

$$\frac{n+1}{4\pi G n}\frac{K}{r^2}\frac{d}{dr}\left(r^2 \rho^{\frac{1-n}{n}}\frac{d\rho}{dr}\right) = -\rho$$

SOLUTION $\rho(r)$ IS POLYTROPE OF INDEX n

B.C: $\rho = 0$ @ $r = R$ because $P(R) = 0$

$\frac{dP}{dr} = 0$ @ $r = 0$ since $\left.\frac{d\rho}{dr}\right|_0 = 0$

POLYTROPE UNIQUELY DEFINED BY K, n, R

$\rightarrow$ COMPUTE P, m functions, e.g

LANE EMDEN EQUATION

INTRODUCE DIMENSIONLESS VARIABLE Θ WITH:

$0 \leq \Theta \leq 1$ such that $\rho = \rho_c \cdot \Theta^n$ ← function

$$\frac{n+1}{n} \frac{K}{4\pi G} \frac{1}{r^2} \frac{d}{dr}\left(\rho_c^{\frac{1-n}{n}} \Theta^{1-n} r^2 \frac{d \rho_c \Theta^n}{dr}\right) = -\rho_c \Theta$$

$$\underbrace{\left[\frac{(n+1) \cdot K}{4\pi G \, \rho_c^{\frac{n-1}{n}}} \right]}_{=: \alpha^2} \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{d\Theta}{dr}\right) = -\Theta^n$$

$$\alpha = \sqrt{\frac{(n+1) \cdot K}{4\pi G \, \rho_c^{\frac{(n-1)}{n}}}}$$

DEF: $r := \alpha \cdot \xi$ ← SCALED RADIUS VARIABLE

$$\alpha^2 \frac{1}{(\alpha \xi)^2} \frac{d}{d\alpha\xi}\left((\alpha \xi)^2 \frac{d\Theta}{d(\alpha \xi)}\right) = -\Theta^n$$

$$\boxed{\frac{1}{\xi^2} \frac{d}{d\xi}\left(\xi^2 \frac{d\Theta}{d\xi}\right) = -\Theta^n}$$

LANE EMDEN EQUN.

→ ONLY DEPENDS ON n

WITH BOUNDARY CONDITIONS: $\Theta(\xi=0) = 1$, $\left.\frac{d\Theta}{d\xi}\right|_{\xi=0} = 0$

→ BOTH INNER BOUNDARY CONDITIONS

→ CAN INTEGRATE INWARD OUT

→ H/W (TRICK: CONVERT TO 1^{ST} ORD ODE SYSTEM)

SOLUTIONS TO LANE EMDEN EQUN
(POLYTROPIC STELLAR MODELS)

RADIUS OF STAR

$$\boxed{R = \alpha \cdot \xi_1}$$

$\Theta(\xi_1) \overset{!}{=} 0$ DEFINES ξ_1 "ONE"

MASS OF STAR:

$$M = \int_0^R 4\pi r^2 \rho \, dr = 4\pi \alpha^3 \int_0^{\xi_1} \xi^2 \Theta^n \, d\xi \quad \Bigg| \quad \begin{array}{l} r = \alpha \cdot \xi \\ \rho = \rho_c \Theta^n \end{array}$$

USE LE:

$$\xi^2 \Theta^n = - \frac{d}{d\xi}\left(\xi^2 \frac{d\Theta}{d\xi} \right)$$

$$M = -4\pi \alpha^3 \rho_c \int_0^{\xi_1} \frac{d}{d\xi}\left(\xi^2 \frac{d\Theta}{d\xi} \right) d\xi$$

$$= -4\pi \alpha^3 \rho_c \, \xi_1^2 \left(\frac{d\Theta}{d\xi} \right)_{\xi_1}$$

DEF. $M_n := - \xi_1^2 \left(\frac{d\Theta}{d\xi} \right)\Big|_{\xi_1} > 0$ WITH ξ_1 AND $\frac{d\Theta}{d\xi}\Big|_{\xi_1}$ FROM SLN OF LE

$$R_n := \xi_1$$

$\rightarrow$

$$\boxed{\begin{array}{l} M = 4\pi \alpha^3 \rho_c M_n \\ R = \alpha \cdot R_n \end{array}}$$