

BASICS OF NUCLEAR REACTION (NETWORKS)

NOTATIONS:

$$Y_i = \frac{X_i}{A_i}, \quad A_i = N_i + Z_i$$

MORE ACCURATELY:

need to take into account mass excess, but for PRACTICAL REASONS, this is usually not done.

$$Y_i = \frac{X_i}{m_i / u}$$

atomic mass unit $\frac{1}{12} m(^{12}\text{C})$
 m_i : mass of nucleus / neutral atom

Except where we need Δmc^2 for Q-VALUES (Energy released by reaction)

$$X_i = \frac{S_i}{S} \quad \boxed{\text{RECALL} \quad \frac{\partial X_i}{\partial t} = f_i(S, T, \vec{X})} \quad \begin{matrix} A \\ Z \end{matrix} \begin{matrix} E \\ N \end{matrix}$$

$$n_i = \frac{S_i}{A_i u} \quad u = \frac{1}{12} u(^{12}\text{C}) \quad \underline{\text{BY DEFINITION}}$$

$$u = 1.6605402 \times 10^{-24} \text{ g}$$

$$Y_i = \frac{S_i}{S \cdot A_i} \quad ; \quad n_i = \frac{S_i}{u \cdot A_i} \rightarrow X_i = n_i \cdot \frac{A_i}{S} \cdot u$$

REACTIONS:

- CONSERVE # nucleons

$$\sum_{\text{IN}} A_i = \sum_{\text{OUT}} A_i$$

- CONSERVE TOTAL CHARGE

$$\sum_{\text{IN}} z_i = \sum_{\text{OUT}} z_i$$

- CONSERVE LEPTON #

$$\sum_{\text{IN}} l_i = \sum_{\text{OUT}} l_i$$

IN PRINCIPLE:



(OR VECTOR NOTATION WITH $\underline{1}, \underline{2}, \dots$ "SPECIES UNIT VECTORS")

- GO OVER ALL SPECIES

- FOR EACH REACTION

" α_x ", " β_x "

FOR SPECIES NOT INVOLVED IS 0

- SUM IT ALL UP

EXAMPLE



$$A: \quad 13 + 4 \quad = \quad 16 + 1 \quad = 17 \checkmark$$

$$Z: \quad 6 + 2 \quad = \quad 8 + 0 \quad = 8 \checkmark$$

$$l: \quad 0 \quad = \quad 0 \quad = 0 \checkmark$$

$\begin{matrix} \rightarrow \\ \leftarrow \end{matrix}$ CROSS SECTION

→ rate proportional

VELOCITY × CROSS SECTION × DENSITY

(FOR BINARY REACTIONS)

$$R \sim \sigma \cdot v$$

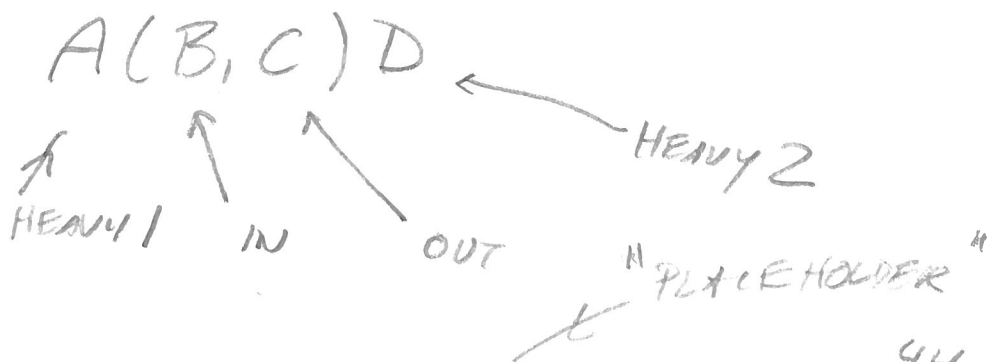
LET US WRITE

$$R_{\alpha_1 1 + \alpha_2 2 + \dots} \rightarrow \beta_1 1 + \beta_2 2 + \dots$$



REACTION NOTATION

BINARY / SINGLE REACTIONS



IN, OUT: $\gamma, n, p, \alpha, {}^4\text{He}$

γ ← photon



could be $\nu_e, \nu_e + e^+, \bar{\nu}_e, \bar{\nu}_e + e^+$
 e^-, e^+

ALSO $\beta^- = ([\gamma^-] e^- \bar{\nu}_e)$

$\beta^+ = ([\gamma^+] e^+ \nu_e)$

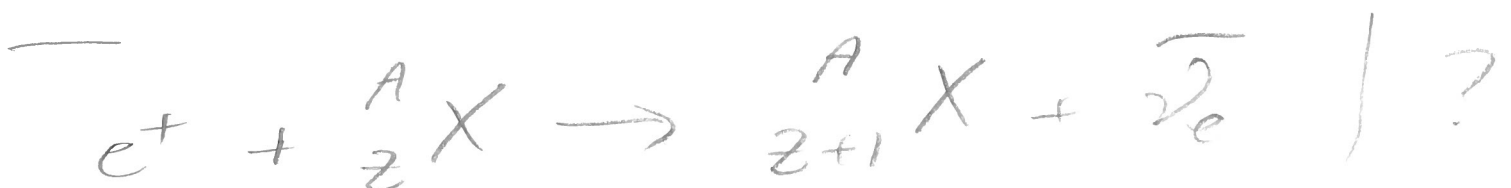
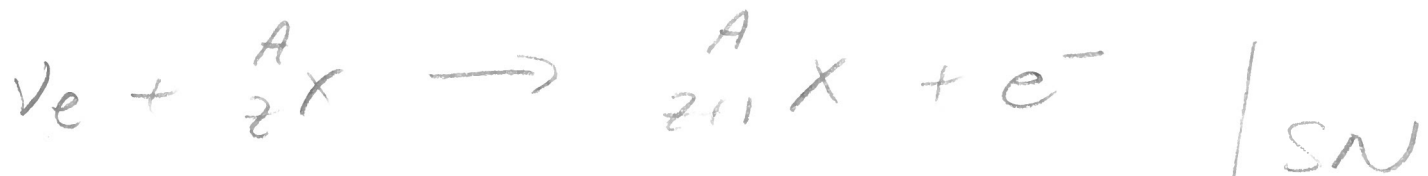
EC = (e^-, ν_e)

LEPTON# CONSERVATION - WEAK REACTIONS

$$\sum_{IN} l_i = \sum_{OUT} l_i$$

Here: mostly electron-lepton#

CASES (typical)



How does this change X_i ?

Need $\frac{\partial}{\partial t} X_i$

USE $Y_i = \frac{1}{A_i} X_i$

$$\rightarrow \frac{\partial}{\partial t} X_i = \frac{1}{A_i} \frac{\partial}{\partial t} Y_i$$

\rightarrow compute Y_i !

• Y_i will decrease for particles used by reaction ($n \rightarrow v$) $\frac{\partial}{\partial t} Y_i < 0$

• Y_i will increase for particles produced by reaction $\frac{\partial}{\partial t} Y_i > 0$

• For Identical particles we have to divide by # permutations to avoid double counting
 $\rightarrow \frac{1}{\alpha_i!}$ — but only on input channel

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1, \alpha_2, \dots \\ \beta_1, \beta_2, \dots}} \lambda_{\alpha_1, 1 + \alpha_2, 2 + \dots} \rightarrow \beta_1, 1, \beta_2, 2, \dots \times \frac{\beta_i - \alpha_i}{\alpha_1! \alpha_2! \dots} Y_1^{\alpha_1} Y_2^{\alpha_2} \dots$$

\sum ALL REACTIONS $(\beta_i - \alpha_i)! \prod_j (Y_j^{\alpha_j} / \alpha_j!)$

MASS EXCESS

$$\Delta m_i = (m_i - A_i \cdot u) c^2$$

measure in MeV [usually]

$$1 \text{ MeV} = 1.6021772 \times 10^{-6} \text{ J}$$

$$u = \frac{1}{12} m(^{12}\text{C}) \rightarrow \Delta m_{^{12}\text{C}} = 0$$

→ WEIGHT EXCESS OF NEUTRAL ATOM RELATIVE TO $\frac{1}{12} m(^{12}\text{C}) \times A_i$

NUCLEAR BINDING ENERGY

Energy needed to separate nuclei

Q: When does this make a difference?

→ weak decays

$n \leftrightarrow p$ conversion

$$m_n > m_p + m_e$$

$$m_n = 939.5656 \text{ MeV}$$

$$m_p = 938.2723 \text{ MeV}$$

$$m_e = 0.510999 \text{ MeV}$$

$$Q_{n \rightarrow p + e^-} = 0.7823 \text{ MeV}$$

ENERGY FROM NUCLEAR REACTIONS

$$E = \Delta m c^2$$

$$Q = \left(\sum_{\text{OUT}} m_i - \sum_{\text{IN}} m_i \right) c^2$$

"PRODUCTS" "REACTANTS"

NOTE:
INCLUDE e^-
(+E CARRIED BY ν_e)

"Q-VALUE" OF REACTION

$$E_{\text{NUC, REACTION}} \sim \frac{dQ}{dt} \Big|_{\text{REACTION}}$$

$$= \lambda_{\alpha,1} + \lambda_{\alpha,2} \rightarrow \beta_1 + \beta_2 + \frac{1}{\lambda_i} Q_{\alpha,1+\alpha,2 \rightarrow \dots}$$

$$E_{\text{TOT}} = \sum_{\text{[ALL REACTIONS]}} = \sum_{\text{[}\alpha_1, \alpha_2, \dots \text{]}} \text{[}\beta_1, \beta_2, \dots \text{]}$$

OR

$$Q = \left(\sum_{\text{OUT}} \Delta m_i - \sum_{\text{IN}} \Delta m_i \right)$$

mass EXCESS
(includes c^2 by def)

INCLUDE $m_e \pm$
 $m_\nu \ll m_e$

+ E_ν carried away
(usually γ)

GENERAL

$$E_{\text{nuc}} = \sum_{\text{REACTIONS}} q_i^{\text{nuc}} = - \frac{d}{dt} \sum_{\text{NUCLEI}} Y_i \Delta m_i$$

$$E_{\text{NIC}} = - \sum_i \Delta m_i \frac{dY_i}{dt}$$

(BUT NEED TO
ADD E_V
FOR WEAK
REACTIONS)

→ RATE OF CHANGE OF MASS EXCESS
OF ALL NUCLEI (include e^-) $\times (-c^2)$

↳ ENERGY RELEASE RATE
= - RATE OF CHANGE
OF MASS EXCESS

("MASS EXCESS" = "MASS DIFFERENCE
FROM $A_1 u \times c^2$)

MORE EXAMPLES



How many "Changes" $\frac{\partial Y_i}{\partial t}$ EQUATION?



NOTE



→ 3 body out

→ CONTINUOUS ν spectrum



→ ONLY 2 body out

→ LINE ν SPECTRUM

NOTE ON "S" (density) dependence of REACTION RATE λ

FOR STRONG REACTIONS

$$\lambda \sim S^{\sum \alpha_i - 1}$$

FOR WEAK REACTIONS. the conc of EC is different in the sense that e^- reacts as per α_i in the nn channel

${}^7\text{Be} \xrightarrow{\text{EC}} {}^7\text{Li} : e^- + {}^7\text{Be} \rightarrow {}^7\text{Li} + \nu_e$

density dependence also comes from degeneracy in case of high density $\rightarrow E_F$
 + there can be "blocking" of e^- in \rightarrow these \rightarrow holes
 the out channel as new e^- needs to have at least E_F

Exercise

COMPUTE Q VALUE OF n DECAY:

$$m_n = 939.5656 \text{ MeV}$$

$$m_p = 938.2723 \text{ MeV}$$

$$m_e = 0.511999 \text{ MeV}$$

AND ENERGY GENERATION RATE

$$T_{1/2} = 10.25 \text{ min} \\ (615 \text{ s})$$

$$Y(t) = Y_0 e^{-\lambda t} \rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}} / \ln$$

$$\ln 2 = \lambda T_{1/2}$$

$$\rightarrow \lambda = \frac{\ln 2}{615 \text{ s}} = 1127 \times 10^{-3}$$

$$E_T = Q_T \cdot \frac{u}{m_n} \cdot N_A = 8.817 \times 10^{-4} \text{ MeV/s} \cdot N_A \cdot \frac{u}{m_n}$$

$$1 \text{ eV} = 1.60217 \times 10^{-12} \text{ J}$$

$$E_T = 1.4026 \times 10^{-9} \cdot N_A \frac{\text{eV}}{\text{s}} \cdot \frac{u}{m_n} \quad \left| \begin{array}{l} N_A = 6.022 \times 10^{23} \\ m_n = 1.00866494 \end{array} \right.$$
$$= 8.4306 \times 10^{14} \text{ eV/g} \quad \left| \begin{array}{l} \text{but } \bar{\nu}_e \text{ carries away} \\ \text{ABOUT } \frac{1}{3} \text{ OF } E \end{array} \right.$$