BASICS OF NUCLEAR REACTION (NETWORKS)

NOTATIONS:

$$Y_i = X_i / A_i \quad , \quad A_i = N_i + Z_i$$

MORE ACCURATELY:
need to take into account mass excess, but for PRACTICAL REASONS, this is usually not done.

$$Y_i = \frac{X_i}{m_i/u}$$

atomic mass unit $\frac{1}{12} m(^{12}C$

m_i: mass of nucleus / neutral atom

Except where we need Δm for Q-VALUES (Energie released by reaction)

$$X_i = \frac{\rho_i}{\rho}$$

RECALL

$$\boxed{\frac{\partial X_i}{\partial t} = f_i(\rho, T, \vec{X})}$$

$${}^{A}_{Z}E_{N}$$

$$n_i = \frac{\rho_i}{A_i u} \qquad u = \frac{1}{12} m(^{12}C)$$

BY DEFINITION

$$u = 1.6605402 \times 10^{-24} g$$

$$Y_i = \frac{\rho_i}{\rho \cdot A_i} \quad ; \quad n_i = \frac{\rho}{u} \frac{X_i}{A_i} \Rightarrow X_i = n_i \frac{A_i}{\rho} \cdot u$$

REACTIONS:

- CONSERVE # nucleons

$$\sum_{IN} A_i = \sum_{OUT} A_i$$

- CONSERVE TOTAL CHARGE

$$\sum_{IN} Z_i = \sum_{OUT} Z_i$$

- CONSERVE LEPTON #

$$\sum_{IN} \ell_i = \sum_{OUT} \ell_i$$

IN PRINCIPLE:

$$\alpha_1 \underline{1} + \alpha_2 \underline{2} + \ldots \longrightarrow \beta_1 \underline{1} + \beta_2 \underline{2} + \ldots$$

(OR VECTOR NOTATION WITH $\underline{1}$, $\underline{2}$, ... "SPECIES UNIT VECTORS")

- GO OVER ALL SPECIES
- FOR EACH REACTION "α_x", "β_x" FOR SPECIES NOT INVOLVED IS 0
- SUM IT ALL UP

EXAMPLE

$$^{13}C + {}^{4}He \longrightarrow {}^{16}O + {}^{1}n$$

A: $13 + 4 = 16 + 1 = 17$ ✓

Z: $6 + 2 = 8 + 0 = 8$ ✓

ℓ: $0 = 0 = 0$ ✓

$\sigma \rightleftarrows O$ CROSS SECTION

→ rate proportional

VELOCITY × CROSS SECTION × DENSITY

(FOR BINARY REACTIONS)

$$\lambda \sim \sigma \cdot v$$

LET US WRITE

$$\lambda_{\alpha_1 1 + \alpha_2 2 + \ldots \rightarrow \beta_1 1 + \beta_2 2 + \ldots}$$

E.G. $\lambda_{^{13}C + \alpha \rightarrow {}^{16}O + n}$

REACTION NOTATION

BINARY / SINGLE REACTIONS

$$A(B, C)D$$

HEAVY 1 IN OUT HEAVY 2

"PLACEHOLDER"

IN, OUT γ, n, p, α ^4He

photon

could be ν_e, $\nu_e + e^+$, $\bar{\nu}_e$, $\bar{\nu}_e + e^+$ $\boxed{\text{out}}$

e^-, e^+

ALSO $\beta^- = ([\text{"}\gamma\text{"}] e^- \bar{\nu}_e)$

$\beta^+ = ([\text{"}\gamma\text{"}] e^+ \bar{\nu}_e)$

$EC = (e^-, \nu_e)$

LEPTON# CONSERVATION — WEAK REACTIONS

$$\sum_{IN} l_i = \sum_{OUT} l_i$$

Here : mostly electron-lepton #

CASES (typical)

$$e^- + {}^A_Z X \longrightarrow {}^A_{Z-1} X + \nu_e \quad (EC)$$

$${}^A_Z X \longrightarrow {}^A_{Z-1} X + e^+ + \nu_e \quad (\beta^+)$$

$${}^A_Z X \longrightarrow {}^A_{Z+1} X + e^- + \bar{\nu}_e \quad (\beta^-)$$

$$\nu_e + {}^A_Z X \longrightarrow {}^A_{Z+1} X + e^-$$

$$\bar{\nu}_e + {}^A_Z X \longrightarrow {}^A_{Z-1} X + e^+$$

SN

$$e^+ + {}^A_Z X \longrightarrow {}^A_{Z+1} X + \bar{\nu}_e$$

?

how does this change X_i?

Need $\frac{\partial}{\partial t} X_i$

USE $Y_i = \frac{1}{A_i} X_i$

$$\rightarrow \frac{\partial}{\partial t} X_i = \frac{1}{A_i} \frac{\partial}{\partial t} Y_i$$

$\rightarrow$ compute Y_i!

- Y_i will decrease for particles <u>used</u> by reaction ("−") $\frac{\partial}{\partial t} Y_i < 0$
- Y_i will increase for particles <u>produced</u> by reaction $\frac{\partial}{\partial t} Y_i > 0$
- For <u>Identical</u> particles we have to divide by # permutations to avoid double counting

$\rightarrow \frac{1}{\alpha_i!}$ — but <u>only</u> on <u>input</u> channel

---

$$\frac{\partial}{\partial t} Y_i = \underbrace{\sum_{\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}}}_{\sum \text{ ALL REACTIONS}} \lambda_{\alpha_1 \underline{1} + \alpha_2 \underline{2} + \ldots \rightarrow \beta_1 \underline{1} + \beta_2 \underline{2} + \ldots} \times \underbrace{\frac{\beta_i - \alpha_i}{\alpha_1! \alpha_2! \ldots} Y_1^{\alpha_1} Y_2^{\alpha_2}}_{(\beta_i - \alpha_i) \prod_j \left(Y_j^{\alpha_j} / \alpha_j! \right)}$$

MASS EXCESS

$$\Delta m_i = (m_i - A_i \cdot u)\, c^2$$

measure in MeV [usually]

1 MeV = 1.6021772×10^{-6} erg

$u = \frac{1}{12} m(^{12}C) \longrightarrow \Delta m_{^{12}C} = 0$

→ WEIGHT EXCESS OF NEUTRAL ATOM RELATIVE TO $\frac{1}{12}(^{12}C) \times A_i$

NUCLEAR BINDING ENERGY

Energy needed to separate nuclei

Q: When does this make a difference?

→ weak DECAYS

$n \leftrightarrow p$ CONVERSION

$m_n > m_p + m_e$

$m_n = 939.5656$ MeV
$m_p = 938.2723$ MeV
$m_e = 0.510999$ MeV

$Q_{n \to p + e^-} = 0.7823$ MeV

ENERGY FROM NUCLEAR REATIONS

$$E = \Delta m \, c^2$$

$$Q = \Big(\underbrace{\sum_{\text{OUT}} m_i}_{\text{"PRODUCTS"}} - \underbrace{\sum_{\text{IN}} m_i}_{\text{"REACTANTS"}} \Big) c^2$$

NOTE: INCLUDE e^-

(+ E CARRIED BY ν_e

"Q - VALUE" OF REATION

$$\mathcal{E}_{\text{NUC, REACTION}} \sim \left. \frac{dQ}{dt} \right|_{\text{"REACTION}}$$

$$= \lambda_{\alpha_1 1 + \alpha_2 2 \to \beta_1 1 + \beta_2 2 + \ldots} \frac{1}{\alpha_1} Q_{\alpha_1 1 + \alpha_2 2 \to \ldots}$$

$$\mathcal{E}_{\text{TOT}} = \sum_{\left[\substack{\text{ALL} \\ \text{REATIONS}}\right]} = \sum_{\left[\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}\right]}$$

$\boxed{\text{OR}}$

$$Q = \Big(\sum_{\text{OUT}} \Delta m_i - \sum_{\text{IN}} \Delta m_i \Big)$$

mass Excess (includes c^2 by def)

INCLUDE $m_{e^\pm}$

$m_\nu \ll m_e$

+ E_ν carried away (usually $\frac{1}{3}$)

GENERAL

$$\varepsilon_{nuc} = \sum_{\text{REACTIONS}} q_i^{nuc} = -\frac{\partial}{\partial t} \sum_{\text{NUCLEI}} Y_i \, \Delta m_i$$

$$\boxed{\varepsilon_{nuc} = -\sum_i \Delta m_i \frac{\partial}{\partial t} Y_i}$$

(BUT NEED TO ADD ε_ν FOR WEAK REACTIONS)

→ RATE OF CHANGE OF MASS EXCESS OF ALL NUCLEI (include e^-) $\times (-c^2)$

↔ ENERGY RELEASE RATE = − RATE OF CHANGE OF MASS EXCESS

("MASS EXCESS" = "MASS DIFFERENCE FROM $A_i u$ $\times c^2$"]

MORE EXAMPLES

$^{13}C + \alpha \rightarrow {}^{16}O + n$

$^{13}C(\alpha, n)\,{}^{16}O$

How many "changes" $\frac{\partial}{\partial t} Y_i$ EQUN?

3α : $3\,{}^{4}He \rightarrow {}^{12}C$

$^{9}B \rightarrow \alpha\alpha p$

$^{13}N(\beta^+)\,{}^{13}C + e^+ + \nu_e$

NOTE

$^{13}N(\gamma, e^+\nu)^{13}C$ | $^{13}N(\beta^+)^{13}C$

→ 3 body out

→ CONTINUOUS ν spectrum

$^7Be(EC)\ ^7Li$ $^7Be(e^-, \nu_e)^7Li$

→ ONLY 2 body OUT

→ LINE ν SPECTRUM

NOTE ON "ρ" [density] dependence of REACTION RATE λ

FOR STRONG REACTIONS

$$\lambda \sim \rho^{\sum \alpha_i - 1}$$

FOR WEAK REACTIONS. the case of EC is different in the sense that e^- counts as an "α_i" in the in channel

$^7Be \xrightarrow{EC} {}^7Li$: $e^- + {}^7Be \to {}^7Li\, \nu_e$

density dependence also comes from degeneracy in case of high density $\to E_F$ rises $\to$ hotter

+ there can be "blocking" of e^- in the out channel as new e^- needs to have at least E_F

Excersise

COMPUTE Q VALUE OF n DECAY:

$m_n = 939.5656$ MeV
$m_p = 938.2723$ MeV
$m_e = 0.511999$ MeV

AND ENERGY GENERATION RATE

$\tau_{1/2} \approx 10.25$ min
(615 s)

$Y(t) = Y_0 e^{-\lambda t} \rightarrow \frac{1}{2} = e^{-\lambda \tau_{1/2}} \Big| \ln$

$\ln 2 = \lambda \tau_{1/2}$

$\rightarrow \lambda = \frac{\ln 2}{615\,s} = 1.127 \times 10^{-3}$

$\varepsilon_T = Q_T \cdot \frac{u}{m_n} \cdot N_A = 8.817 \times 10^{-4}$ MeV/s $\cdot N_A \cdot \frac{u}{m_n}$

$1 eV = 1.60217 \times 10^{-12}$ erg

$\varepsilon_T = 1.4126 \times 10^{-9} \cdot N_A \frac{erg}{s} \cdot \frac{u}{m_n}$ | $N_A = 6.022 \times 10^{23}$
$m_n = 1.0086649 u$

$= 8.4306 \times 10^{14}$ erg/g | but $\overline{\nu_e}$ carries away ABOUT 1/3 OF E