TOTAL ENERGY OF STAR

- INTERNAL E: $U = \int_0^M u \, dm$
- POTENTIAL E: $\Omega = -\alpha \cdot \frac{GM^2}{R}$
- KINETIC E: $K = \int_0^M \frac{1}{2}\left(\frac{\partial}{\partial t} r\right)^2 dm$

→ TOTAL $E_{TOT} = U + K + \Omega$

SINKS & LOSSES:

- LOSS @ SURFACE $L > 0$
- INPUT FROM BURNING $L_{nuc} = \int_0^M \varepsilon_{nuc} \, dm$
- NEUTRINO LOSSES $L_\nu = \int_0^M \varepsilon_\nu \, dm$

$L_{TOT} = -L + L_{nuc} - L_\nu$

→ ENERGY CONSERVATION

$$\frac{d}{dt} E_{TOT} = L_{TOT}$$

BINDING ENERGY

$$\Omega = -\alpha \frac{GM^2}{R}$$

LIMITS ON α?

$$\int_0^M \frac{Gm}{r}\,dm = \int_0^M \frac{Gm}{r(m)}\,dm \geq \int_0^M \frac{Gm}{R}\,dm = \frac{1}{2}\frac{GM^2}{R}$$

$$\rightarrow \alpha \geq \frac{1}{2}$$

FOR REAL STARS: (E.G. RED GIANTS)

α can be $\gg 1$!

TIME SCALES OF STELLAR EVOLUTION

$$\tau = \phi / \dot{\phi} = \frac{dt}{d\ln\phi}$$

1) DYNAMIC TIME SCALE

$\Leftrightarrow$ SPATIAL (RADIUS) CHANGE

$$\dot{R} \sim v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$\rightarrow \dot{R}/R = \sqrt{R^3/2GM} = \tau_{DYN}$$

USE $M = \frac{4\pi R^3}{3} \bar{\rho}$

$$\rightarrow \tau_{DYN} = \sqrt{\frac{3}{8\pi \bar{\rho} G}} \simeq \frac{1}{\sqrt{G\bar{\rho}}}$$

$$\approx 10^3 s \sqrt{\left(\frac{R}{R_\odot}\right)^3 \left(\frac{M_\odot}{M}\right)}$$

2) THERMAL TIME SCALE (GRAVO- THERMAL)

$$\tau_{KH} = \frac{U}{L} = \alpha \cdot \frac{GM^2}{RL}$$

α ~ 1/2 (typical)

$$\boxed{\text{KELVIN-HELMHOLTZ TIME SCALE}}$$

$\frac{GM^2}{2RL}$ I USUALLY USE

3) NUCLEAR TIME SCALE

$$E_{nuc} = \varepsilon \cdot M c^2$$

10^{21} erg/g

CONSIDER:
- ENERGY RELEASE PER NUCLEON (5 MeV/nuc FOR $X_\odot$
- FRACTION OF STAR THAT BURNS

$$\tau_{nuc} = \frac{E_{nuc}}{L} \simeq \varepsilon \cdot 10^{21} \cdot s \left(\frac{M}{L} \cdot \frac{erg/s}{g} \right)$$

$$\boxed{\varepsilon \sim 10^{-3} \frac{E_{nuc}}{1\,MeV}}$$

Virial Theorem

Separation of variables

$$dP = -\frac{Gm}{4\pi r^4}\,dm \quad \Bigg| \quad V = \frac{4\pi}{3} r^3 \quad \Bigg| \quad \int_{\text{centr}}^{\text{surf}}$$

$$\int_{0 \rightsquigarrow P_c}^{P(R)} V\,dP = \underbrace{-\frac{1}{3}\int_0^M \frac{Gm}{r}\,dm}$$

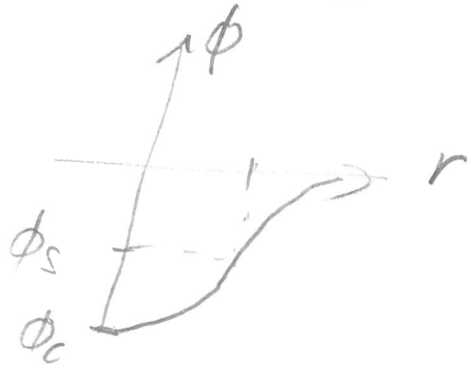
$$\boxed{\Omega = -\int_0^M \frac{Gm}{r}\,dm}$$

Total potential energy of star

Q: How to compute? General formula for Ω?

$$-\frac{1}{2}\int \frac{G\,\rho(\vec r_1)\,\rho(\vec r_2)}{|\vec r_1 - \vec r_2|}\,dV_1\,dV_2$$

$$\frac{1}{2}\int \phi(r)\,\rho(r)\,dV$$



$$\int_{P_c}^{P(R)} V\,dP = \left[PV\right]_0^{R} - \int_0^{V(R)} P\,dV = -\int_0^{M} \frac{P}{\rho}\,dm$$

$=0$ @ SURF

CENTER $\rightarrow R=V=0$

$$P_C V_C - P_{SURF} V_{SURF}$$

$0 \qquad 0$

$$dV = 4\pi r^2 dr$$
$$dm = 4\pi r^2 \rho\, dr$$
$$dm = \rho(m)\, dV$$

$$\rightarrow \frac{1}{3}\Omega = -\int_0^{M} \frac{P}{\rho}\,dm$$

"PARTIAL" VIRIAL THEOREM (STAR):

$\rightarrow$ SUPPOSE CORE W/ ENVELOPE, AS EXAMPLE

$\rightarrow P_{SURF} V_{SURF} \neq 0 \quad (P_{SURF} \neq 0)$

$$P_S V_S - \int_0^{M_S} \frac{P}{\rho}\,dm = \frac{1}{3}\Omega_S$$

$$\text{with } \Omega_S = -\int_0^{M_S} \frac{Gm}{r}\,dm$$

Ideal gas

$$P = \frac{\mathcal{R} T \rho}{\mu}$$

GAS CONSTANT

$\mathcal{R} = k_B / u$

$= k_B \cdot N_A$

$$u = \frac{3}{2} \frac{\mathcal{R} T}{\mu} = \frac{3}{2} \frac{P}{\rho}$$

(internal energy per unit mass
$\Leftrightarrow$ specific internal energy)

$\rightarrow$ TOTAL INTERNAL ENERGY OF STAR:

$$U = \int_0^M u \, dm = \frac{3}{2} \int_0^M \frac{P}{\rho} \, dm$$

ON THE OTHER HAND: VIRIAL THEOREM:

$$-\frac{1}{3} \Omega = \int_0^M \frac{P}{\rho} \, dm = \frac{2}{3} U$$

$$\Rightarrow U = -\frac{1}{2} \Omega$$

Q: REL DEG GAS? $\quad u = 3 \frac{P}{\rho}$

$\rightarrow$ ... WHAT DO WE FIND?
WHAT DOES IT MEAN?

HEAT CAPACITY OF STARS

IDEAL GAS: $U = -\frac{1}{2}\Omega$

HYDROSTATIC : $K = 0$

$U > 0$

$\rightarrow E = U + \Omega = \frac{1}{2}\Omega = -U < 0$

ASSUME AVERAGE TEMPERATURE $\bar{T} \sim U > 0$

$$C = \frac{dE}{dT} \simeq -\frac{dU}{d\bar{T}} < 0$$

→ STARS HAVE NEGATIVE HEAT CAPACITY

Q: WHAT DOES THIS MEAN?

→ WHEN YOU EXTRACT E THEY GET HOTTER

Q: WHAT HAPPENS IF YOU COVER STAR WITH INSULATING LAYER?

Q: WHAT HAPPENS IF YOU ADD EXTRA HEAT IN CENTER, SAY... E.G. FASTER NUCLEAR BURNING?

Q: HEAT CAPACITY OF REL. DEG. STAR

AVERAGE T

RANDOM NOTES

RECALL $U = \int_0^M \frac{3}{2} \frac{\mathcal{R} T}{\mu} \, dm := \frac{3}{2} \frac{\mathcal{R} \bar{T}}{\mu} M$

ASSUMING $\mu =$ CONST

$$\bar{T} = \int_0^M T \, dm \Big/ \int_0^M dm$$

IN REALITY: BOTH T, μ ARE FUNCTION OF M

$$\overline{\left(\frac{T}{\mu}\right)} = \int_0^M \frac{T}{\mu} \, dm \Big/ \int_0^M dm$$

$$\rightarrow U = \frac{3}{2} \mathcal{R} \overline{\left(\frac{T}{\mu}\right)} M$$

$$\Omega = -\alpha \frac{GM^2}{R} = \frac{1}{2} U \Rightarrow \bar{T} = \frac{\alpha}{3} \frac{GM}{R \mathcal{R}}$$

$\boxed{\simeq 4 \times 10^6 \, K \text{ FOR SUN}}$