STELLAR EOS

UNIQUE FUNCTION:

$$P = P(\rho, T, X)$$

COULD ALSO BE SOLVED FOR T, ρ, ...

$$\rho = \rho(P, T, X)$$

$$T = T(P, \rho, X)$$

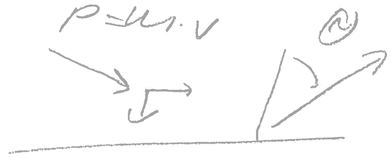
OTHER VARIABLES SOMETIMES USED: S, e ...

IN STARS

ATOMS CAN BE IONIZED (USUALLY)
NUCLEI CAN INTERACT
ELECTRONS CAN BE DEGENERATE
RADIATION CAN CONTRIBUTE
$\hookrightarrow$ e^+e^- PAIR PRODUCTION
$\gamma\gamma \rightarrow e^+e^- \rightarrow \gamma\gamma$
$\nu\nu$

PRESSURE

$p = m \cdot v$



$$\Delta p = 2 \cdot p_\perp = 2p\cos\theta$$

$$P = \frac{1}{3}\int_0^\infty v \cdot p \cdot n(p)\, dp$$

$$P = \underbrace{P_{ions} + P_e}_{P_{GAS}} + P_{RAD}$$

$$P_{GAS} + P_{RAD}$$ ← INCLUDES e^+e^- PAIRS

DEF: $\beta = \frac{P_{GAS}}{P} \Leftrightarrow P_{GAS} = \beta \cdot P$

$$P_{RAD} = (1-\beta) \cdot P$$

OBVIOUSLY: $0 < \beta < 1$

$\beta \ll 1$: radiation-dominated

$\beta \sim 1$: gas-dominated

→ CONSEQ. FOR STABILITY OF STARS

ION PRESSURE

MAXWELL BOLTZMANN DISTRIBUTION:

EACH SPECIES: i

$$n_i(p)\,dp = \frac{4\pi n_i\, p^2\, dp}{\sqrt{2\pi m_i k_B T}^{\,3}}\, e^{-\frac{p^2}{2m_i k_B T}}$$

$$P_i = \frac{1}{3}\int v\, p\, n_i(p)\,dp = n_i k_B T$$

total # density:

$$n_I = \sum_i n_i \quad \sum_i \frac{\rho}{u} Y_i \,, \quad Y_i = \frac{X_i}{A_i}$$

Now:

$$\frac{1}{\mu_I} = \sum_i Y_i \Rightarrow n_I = \frac{\rho}{\mu_I u}$$

APPROX STELLAR GA

$$\frac{1}{\mu_I} = X + \frac{1}{4}Y + (1-X-Y)\frac{1}{\langle A\rangle_{\text{METAL}}}$$

FOR SUN: $\langle A\rangle_Z \simeq 20 \Rightarrow \mu_I \simeq 1.29$

$\Re = k_B N_A$

$$P_I = \sum_i P_i = \frac{\Re T \rho}{\mu_I}$$

IDEAL ELECTRON GAS

SIMILAR TO ION GAS

$$P_e = n_e k_B T \,, \quad n_e = \sum_i Z_i n_i = \frac{\rho}{u} \sum_i Z_i Y_i$$

DEF:

$$\frac{1}{\mu_e} = \sum_i Z_i Y_i \rightarrow n_e = \frac{\rho}{\mu_e \cdot u}$$

FOR STELLAR GAS: MOSTLY H & He:

$$\frac{1}{\mu_e} \simeq X + \frac{1}{2} Y + (1-X-Y) \left\langle \frac{Z}{A} \right\rangle_{metals}$$

FOR MOST STELLAR COMPOSITIONS:

$$\left\langle \frac{Z}{A} \right\rangle_{metal} \simeq \frac{1}{2}$$

$\longrightarrow$ APPROXIMATE $\quad \frac{1}{\mu_e} \simeq \frac{1}{2}(1+X)$

FOR SUN: $\mu_e \simeq 1.17$

Q: WHAT FOR: PURE H
PURE He

$$P_e = \frac{\mathcal{R} T \rho}{\mu_e}$$

→ <u>TOTAL GAS PRESSURE</u> (non-deg e^- gas)

$$P_{GAS} = P_I + P_e = \left(\frac{1}{\mu_I} + \frac{1}{\mu_e}\right) \mathcal{R}_g T = \frac{\mathcal{R}_g T}{\mu}$$

$$\Longleftrightarrow \quad \frac{1}{\mu} = \frac{1}{\mu_e} + \frac{1}{\mu_I}$$

FOR SUN WE HAVE $\mu \simeq 0.61$

DEGENERATE e^- PRESSURE

e^- much lighter than ions $m_e \ll m_I$

→ less momentum for given energy $k_B T$

→ become degenerate first

[IN PRACTICE: IN STARS WE DO NOT HAVE TO DEAL WITH DEG IONS]

- Heisenberg $(\Delta x)^3 (\Delta p)^3 \gtrsim h^3$ $\quad h = 6.626 \times 10^{-27} \frac{cm^2 g}{s}$

+ 2 SPIN ORIENTATIONS

→ FOR COMPLETE DEG. ISOTROPIC e^- gas we have

$$n_e(p)\,dp = \begin{cases} \frac{2}{(\Delta x)^3} = \frac{2}{h^3} 4\pi p^2\,dp & p < p_F \\ 0 & p > p_F \end{cases}$$

→ max (FERMI) momentum ⟷ total # of e^-

$$n_e \stackrel{!}{=} \int_0^{p_F} n_e(p)\,dp \rightarrow p_F = \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3}$$

→ PRESSURE INTEGRAL

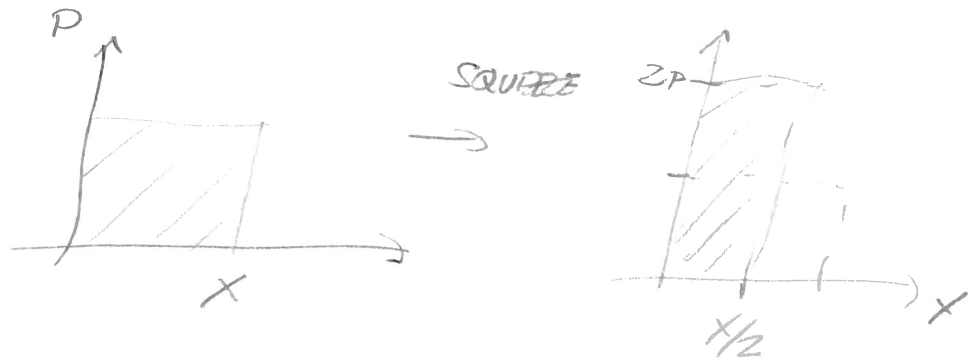
$$P = \frac{1}{3}\int_0^{\infty} v \cdot p \; n(p)\, dp = \frac{1}{3}\int_0^{p_F} v \cdot p \; n(p)\, dp$$

$$= \frac{8\pi p_F^5}{15 m_e h^3} = \frac{h^3}{20 m_e \, u^{5/3}} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{\rho}{\mu_e}\right)^{5/3}$$

$$=: K_1 \cdot \rho^{5/3}$$

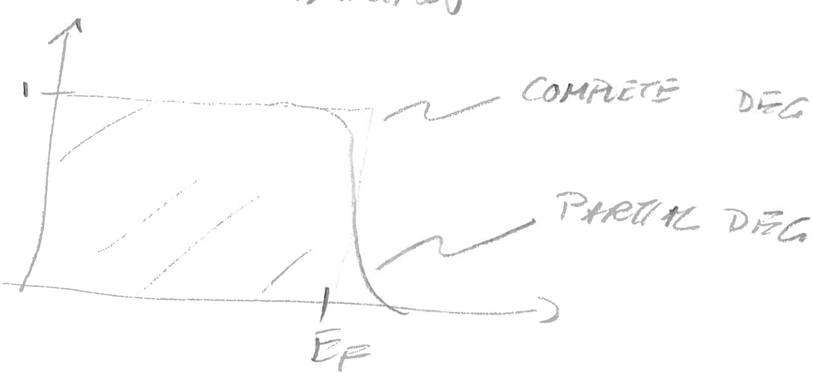
$$n_e = \rho/\mu_e u \quad , \quad K_1 \simeq 10^{13} \frac{\mathrm{dyn\,cm^{-2}}}{(\mathrm{g\,cm^{-3}})^{5/3}}$$

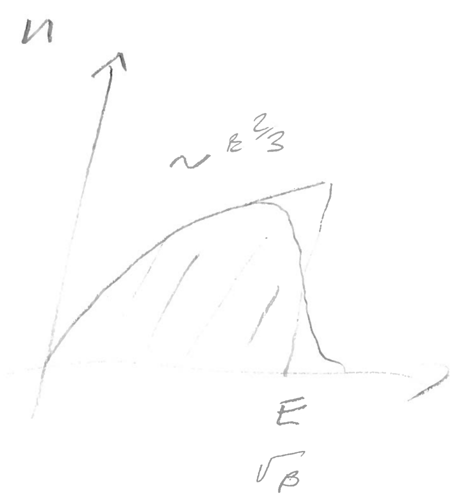
SCHEMATICS



INCOMPRESSIBLE PHASE SPACE

OCCUPATION FRACTION





REL DEG. e^- PRESSURE

GENERALLY: $v = \frac{p}{m_e} \Big/ \sqrt{1 + \frac{p^2}{m_e^2 c^2}}$

but FOR $p \gg m_e c$ we have $v \rightarrow c$

$$P_e = \frac{1}{3} \int_0^{p_F} c \cdot p \cdot n(p)\, dp = \frac{hc}{8u^{4/3}} \left(\frac{3}{\pi}\right)^{1/3} \left(\frac{\rho}{\mu_e}\right)^{4/3}$$

$$=: K_2 \; \rho^{4/3}$$

$$K_2 \sim 1.24 \times 10^{15} \; \frac{\text{dyn cm}^{-2}}{(\text{g cm}^{-3})^{4/3}}$$