RADIATION PRESSURE

PLANCK SPECTRUM $\quad n(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3}\,\frac{1}{e^{\frac{h\nu}{k_BT}}-1}$

$$P_{RAD} = \frac{1}{3}\int_0^\infty c\,\frac{h\nu}{c}\,n(\nu)\,d\nu = \frac{1}{3}aT^4$$

RADIATION CONSTANT $\quad a = \frac{8\pi^5 k_B^4}{15c^3h^3} = \frac{4\sigma}{c}$

COULOMB INTERACTIONS

ASSUME AVERAGE DENSITY ρ, PARTIAL MASS μ

→ PARTICLE DENSITY $n = \rho/(\mu \cdot u)$

→ $\frac{1}{n}$: SPECIFIC VOLUME $\sim d^3$

→ TYPICAL DISTANCE $d \sim n^{-1/3} = \left(\frac{\mu u}{\rho}\right)^{1/3}$

COULOMB ENERGY BETWEEN PARTICLES:

$$e_{coulomb} = \frac{Z^2 e^2}{d}$$

COMPARE TO CHARACTERISTIC ENERGY AT TEMPERATURE T: $k_B T = e_{thermal}$

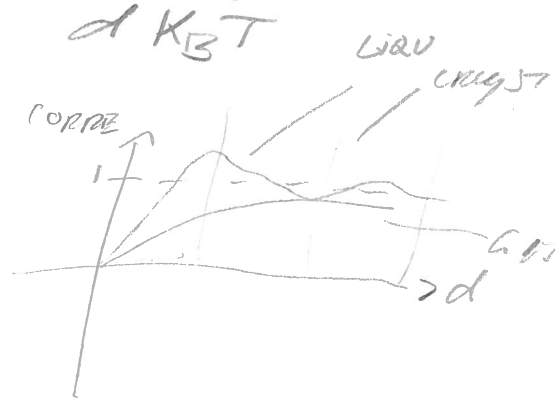
<u>DEF</u>: COULOMB PARAMETER

$$\Gamma := \frac{e_{coulomb}}{e_{thermal}} = \frac{Z^2 e^2}{d\, k_B T}$$

$\Gamma \lesssim 5$: GAS

$5 \lesssim \Gamma \lesssim 175$: LIQUID

$\Gamma \gtrsim 175$: SOLID/CRYSTAL



EXAMPLE

WHAT DO WE HAVE FOR STARS:

$$\rho \rightarrow \bar{\rho} = \frac{3M}{4\pi R^3} \rightarrow d = \left(\frac{4\pi\mu u}{3M}\right)^{1/3} R$$

$$T \rightarrow \bar{T} \simeq \alpha \mu u \, GM / 3k_B R$$

$$\rightarrow \Gamma \sim \frac{3 \, Z^2 e^2}{\mu^{4/3} u^{4/3} \, \alpha \cdot G M^{2/3}} \sim 0.01$$

FOR SUN

Q: DOES THIS MEAN SUN IS NOT DEGENERATE?

INTERNAL ENERGY

$$u = \frac{1}{\rho} \int_0^{\infty} n(p)\, \mathcal{E}(p)\, dp$$

$\mathcal{E}(p) = p \cdot c$

$\uparrow$

REL

CLASSIC

Q: derive

classical gas: $\mathcal{E}(p) = \frac{p^2}{2m_g}$

GENERAL rel. gas: $\mathcal{E}(p) = m_g c^2 \left[\left(1 + \frac{p^2}{2m_g c^2}\right)^{1/2} - 1 \right]$

DEG

$$n(p) = \begin{cases} 8\pi\, p^2/h^3 & p \le p_F \\ 0 & p \ge p_F \end{cases}$$

$$p_F = \left(\frac{3h^3\, n_{gas}}{8\pi}\right)^{1/3}, \quad n_{gas} = \frac{\rho_{gas}}{m_{gas}}$$

NR: $$U_{gas} = \frac{3}{2} \frac{P_{gas}}{\rho}$$

CLASSICAL

$$n(p) = \frac{4\pi n_i p^2}{\sqrt{2\pi m_i T}^{\,3}}\, e^{-\frac{p^2}{2m_i k_B T}}$$

$$\longrightarrow \quad U_{gas} = \frac{3}{2} \frac{P_{gas}}{\rho}$$

REL: $u_{gas} = 3 \frac{P_{gas}}{\rho}$

RAD: $u_{RAD} = \frac{1}{\rho}\int_0^\infty h\nu\, n(\nu)\, d\nu = \frac{1}{\rho}\int_0^\infty \nu h \frac{8\pi\nu^2 d\nu}{c^3} \frac{1}{e^{\frac{h\nu}{k_B T}}-1}$

$$= \frac{a T^4}{\rho} = 3 \frac{P_{rad}}{\rho}$$

$\rightarrow$ SAME!

ADIABATIC EXPONENT $\Leftrightarrow S = \text{const}$

1ST LAW $\quad dq = 0 = du + P\, d\frac{1}{\rho}$

IN MOST CASES $u \sim P/\rho$ with ideal ϕ

$$u = \phi \cdot \frac{P}{\rho}$$

$$\rightarrow \quad 0 = d\left(\phi \frac{P}{\rho}\right) + P\, d\left(\frac{1}{\rho}\right) = \frac{\phi}{\rho}\, dP - (\phi+1) \frac{P}{\rho^2}\, d\rho$$

$$\rightarrow \quad \frac{dP}{d\rho} = \frac{\phi+1}{\phi} \cdot \frac{P}{\rho}$$

DEF (GENERAL) $\quad \gamma_{ad} = \frac{d\ln P}{d\ln \rho} = \frac{\phi+1}{\phi}$ FOR SIMPLE CASES

$$P \sim \rho^{\gamma_{ad}}$$

$\phi \sim n$ $\boxed{\text{ADIABATIC INDEX}}$

→ WHAT DO WE OBTAIN FOR DIFFERENT CASES?

NON-REL CLASSIC/DEG: $\gamma_{ad} = 5/3$

REL DEG / RAD $\quad \gamma_{ad} = 4/3$

EDDINGTON LIMIT

LOOK AT (CLOSE TO) STELLAR SURFACE

FLUX $F = \frac{L}{4\pi r^2}$

ASSUME CONST L IN OUTER LAYER OF STAR

FORCE FROM RAD PRESSURE

$$\frac{dP}{dr} = - \frac{\kappa \rho}{c} \cdot F = \frac{\kappa \rho L}{4\pi c r^2}$$

AND $\frac{dP}{dr} = - \rho \frac{GM}{r^2}$ $\stackrel{!}{=}$

→ SOLVE FOR L : $L_{EDD} = \frac{4\pi c G M}{\kappa}$

SIMPLE APPROXIMATION: CONSIDER JUST e^- SCATTERING

$$K = \kappa_{es} = \frac{\kappa_{es,0}}{\mu_e} \simeq \frac{1}{2} \kappa_{es,0} (1+X)$$

σ_T / u

$$L_{edd} = \underbrace{\frac{4\pi c G u}{\sigma_T}} M$$

$$= 1.3 \times 10^{38} \left(\frac{M}{M_\odot}\right) \sim 3.3 \cdot 10^4 \frac{M}{M_\odot} \cdot L_\odot$$

<u>QUIZ</u>: • DERIVE L_{EDD} FOR PURE He STARS
• EDD ACC RATE

- ASSUME STAR OF MASS M, RAD R
- MATERIAL IS PURE H, fully ionized
- spherical symmetric, free fall from R=∞
- matter hits surf → E_{KIN} → $L_{ACC} \gg L_\odot$

Q: how fast can star accrete at max?
(EDD. ACC. RATE)

EDDINGTON LIMIT NOTES:

- $L_{MAX} \sim M$ → lifetime of star minimal
- for spherical accretion this sets maximum "accretion rate"

assumptions needed:

1) RADIUS OF OBJECT
OR
2) ENERGY RELEASE EFFICIENCY OF ACC.

→ HOW FAST CAN YOU ASSEMBLE ASTROPHYSICAL objects?

But: BREAK OF symmetry (DISK, ...)
OTHER TRANSPORT MODES → convection.

(what would happen in stellar interior IF $L_{SURF} \sim L_{EDD}$?
$m < M$

→ HYDROSTATIC (can SNe be brighter than L_{EDD}? ANY)

→ COMP. DEP. $(1+X)$... κ...