SAHA EQUATION

OUTER LAYERS OF STAR

LET'S CONSIDER ^1H, LOSE $1e^-$, pure H gas

$$H \rightleftharpoons H^+ + e^-$$

n_0 : # density OF neutral H

n_+ : H^+

n_- : e^-

→ total particle density $n = n_+ + n_- + n_0$

⟺ $\rho = (n_+ + n_0) \cdot m_{gas}$

Q: e^-?

NON-REL, NON-DEG. $P = n k_B T$

DEF: DEGREE OF IONIZATION

$$x = \frac{n_+}{n_0 + n_+}$$

NOTE

$$x + 1 = \frac{n_0 + n_- + n_+}{n_0 + n_+} = \frac{n}{n_0 + n_+}$$

$$\rightarrow n = (x+1)(n_0 + n_+)$$

SAHA EQN:

$$\frac{n_+ n_-}{n_0} = \frac{g}{h^3} \left(2\pi m_e k_B T\right)^{3/2} e^{-\chi/k_B T}$$

g: "stat. weight factor

χ ionization potential (13.6 eV)

Ex. II

$\rightarrow$ $P = (1+x)(n_0+n_+) k_B T = (1+x) \rho \mathcal{R} T$

CAN REWRITE AS

$$\frac{x^2}{1+x^2} = \frac{g}{h^3} \frac{(2\pi m_e)^{3/2} (k_B T)^{5/2}}{P} e^{-\chi/k_B T}$$

$\rightarrow$ IONIZATION FRACTION AS FUNCTION OF P, ρ

$x = x(P, \rho)$

$$du + P\, d\left(\tfrac{1}{\rho}\right) = dq = 0 = du - \frac{P}{\rho^2} d\rho$$

$$u = \frac{3}{2} \frac{P}{\rho} + \frac{\chi}{m_{gas}} \cdot x$$

$$\rightarrow du = \frac{3}{2} \frac{dP}{\rho} - \frac{3}{2} \frac{P}{\rho^2} d\rho + \frac{\chi}{m_{gas}} \left(\frac{\partial x}{\partial P} dP + \frac{\partial x}{\partial \rho} d\rho \right)$$

$\times \frac{1}{\rho}$, collect terms...

$$0 = \left[\frac{3}{2} + \frac{\chi}{k_B T}\left(\frac{P}{1+x}\right)\frac{\partial x}{\partial P}\Big)_\rho\right] \frac{dP}{d\rho} - \left[\frac{5}{2} - \frac{\chi}{k_B T}\left(\frac{\rho}{1+x}\right)\frac{\partial x}{\partial \rho}\Big)_P\right] \frac{d\rho}{\rho}$$

$$\rightarrow \gamma_{ad} = \frac{d\ln P}{d\ln \rho} = \frac{5 + \left(\frac{5}{2} + \frac{\chi}{k_B T}\right)^2 x(1-x)}{3 + \left[\frac{3}{2} + \left(\frac{3}{2} + \frac{\chi}{k_B T}\right)^2\right] x(1-x)}$$

FOR $x = 0, 1$ $\quad \gamma_{ad} = 5/3$ as should be

min value for $x = 0$:

Example: $\lambda = 10\, K_B T \longrightarrow \gamma_{ad} = 1.21$