THERMAL INSTABILITY IN DEGENERATE GAS

NR - DEG GAS

$$P \sim P_e = K_1 \rho^{5/3}$$

→ no significant dependence of P on T

$$\left| \left(\frac{\partial \ln P}{\partial \ln T}\right)_\rho \right| \ll 1 \quad \left| \left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P \right| \ll 1$$

except for deviations from complete degeneracy

ASSUME PERTURBATION $+\Delta T$

$T \uparrow \rightarrow \varepsilon_{nuc} \uparrow \rightarrow T \uparrow \rightarrow \ldots$

→ THERMONUCLEAR RUNAWAY

POSSIBLE OUTCOMES:

- → enough fuel is burnt to unbind star
  - → SN (TYPE IA)
- → degeneracy is "lifted" before star is unbound
  - → transition to non-deg EOS
    - → EXPANSION
      - → COOLING

(EXAMPLES: HE SHELL FLASH, XRB, NOVAE

GENERAL THERMAL STABILITY CONDITION.

FROM POLYTROPIC MODEL:

$$\frac{dP_c}{P_c} = \frac{4}{3}\frac{d\rho_c}{\rho_c}$$

ASSUME GENERAL EOS: $\frac{d\rho}{\rho} = \alpha \cdot \frac{dP}{P} - \delta\frac{dT}{T}$

$$\left(\frac{4}{3} - \frac{1}{\alpha}\right)\frac{d\rho_c}{\rho_c} = \frac{\delta}{\alpha}\frac{dT_c}{T_c}$$

$\uparrow > 0$

for non-deg: $\frac{1}{\alpha} < \frac{4}{3}$

→ CONTRACTION ⟷ heating
EXPANSION ⟺ cooling

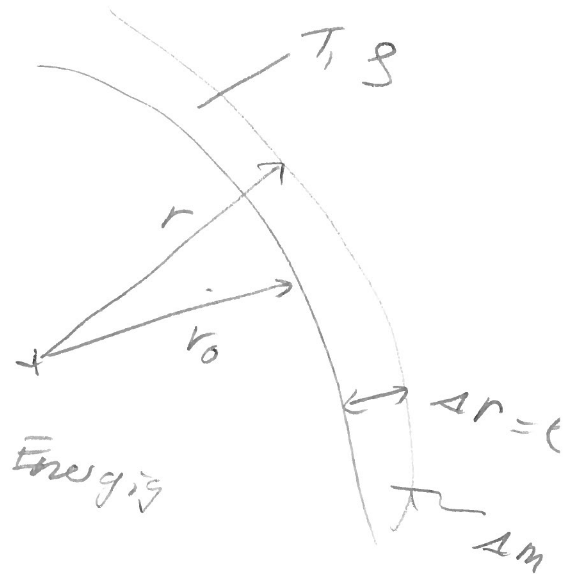
→ STABLE

FOR DEG GAS: $\frac{1}{\alpha} \gtrsim 4/3$

→ expansion ⟷ heating
→ unstable!

THIN SHELL INSTABILITY

$r = r_0 + \Delta r$



$\rightarrow \Delta r = r - r_0 \ll 1$

in thermal equilibrium:
change in $L(r)$ = generated Energy

$$\Delta L = L(r) - L(r_0) = \int_{r_0}^{r} \varepsilon_{nuc} \cdot 4\pi r^2 \rho \, dr$$

SPECIAL

FOR NON-DEG GAS:
EXCESS ENERGY → EXPANSION
TOO LITTLE ENERGY → CONTRACTION

$$\Delta m \simeq 4\pi r_0^2 \Delta r \cdot \rho \rightarrow \frac{d\rho}{\rho} = -\frac{d\Delta r}{\Delta r} = -\frac{dr}{\Delta r} = -\frac{dr}{r}\frac{r}{\Delta r}$$

hydrostatic EQUILIBRIUM: $\frac{dP}{dm} = -Gm/4\pi r^4$

$$\frac{dP}{P} = -4\frac{dr}{r} = 4\frac{\Delta r}{r}\frac{d\rho}{\rho} \quad \Big| \quad \frac{dP}{P} = \frac{1}{\alpha}\frac{d\rho}{\rho} + \frac{\delta}{\alpha}\frac{dT}{T}$$

$$\rightarrow \left(4\frac{\Delta r}{r} - \frac{1}{\alpha}\right)\frac{d\rho}{\rho} = \frac{\delta}{\alpha}\frac{dT}{T}, \quad \frac{\delta}{\alpha} > 0$$

$$\frac{d \ln T}{d \ln \rho} = \frac{\alpha}{\delta}\left(4\frac{\Delta r}{r} - \frac{1}{\alpha}\right) = \frac{1}{\delta}\left(4\alpha\frac{\Delta r}{r} - 1\right)$$

>0 required for stability | FOR THIN SHELL: $\frac{\Delta r}{r} \rightarrow 0$
$\rightarrow \frac{d\ln T}{d \ln \rho} < 0 \rightarrow$ EXP. ⟷ high T

Q: CONSIDER PLANE-PARALLEL APPROX.

(WHAT DOES THIS MEAN?

$g = \text{CONST}$

$P = \text{CONST}$