NUCLEAR MASSES

BINDING ENERGY

$$B(N,Z) = \left(N \cdot m_n + Z \cdot m_p - m(N,Z)\right)c^2$$

→ less than 1% of mass

$$B = -E$$

$$H\psi = E\psi$$

ATOMIC MASSES:

$$m_{AT}(N,Z) = m(N,Z) + Z \cdot m_e - B_{el}(Z)/c^2$$

with

$$B_{el}(Z) \simeq \left(14.4381\, Z^{2.39} + 1.55468 \times 10^{-6}\, Z^{5.35}\right) eV$$

MASS EXCESS: $\Delta M(N,Z) = \left(m_{at}(N,Z) - \frac{A}{12} m_{at}(6,6)\right) \times c^2$

SEMI-EMPERICAL mass FORMULA

WEIZSÄCKER (1935) , BETHE & BACHER (1936)

$$\underline{E} = a_{VOL} \cdot A + a_{SF} \cdot A^{2/3} + \frac{3e^2}{5r_0} Z^2 A^{-1/3}$$

$$+ \left(a_{sym} \cdot A + a_{SS} \cdot A^{2/3}\right) \underline{I}^2$$

$$+ \Delta n + \Delta p$$

$\Delta n, p = \pm \delta$ for N, Z even or odd

[RMS charge RADIUS: $R_{CH} = r_0 \cdot A^{1/3}$

$$\underline{I} = \frac{N - Z}{A}$$

a_{SS} : correction by Myers & SWIATEC (1966)

$a_{VOL} = -15.697550$ MeV

$a_{SF} = 17.662690$ MeV

$a_{sym} = 26.308165$ MeV

$a_{SS} = -17.003132$ MeV

$r_0 = 1.221897$ fm

$\delta = -1.250000$ MeV

$e^2 = 1.43985$ MeV $\cdot$ fm

LARGE-A LIMIT OF WEIZSÄCKER FORMULA

$$e = \frac{E}{A} = a_{vol} + a_{SF} A^{-1/3} + \frac{3e^2}{20 r_0} A^{2/3} (1-I
$$

$$+ \left(a_{sym} + a_{SS} A^{-1/3}\right) I^2$$

AGAIN $I = \frac{N-Z}{A}$

COULOMB TERM: (non-SATURATING)

→ diverge unless $I = 1$

→ PURE neutron system

IF we set $I = 1$:

$$e \sim a_{vol} + a_{sym}$$

→ $\sim +10.6$ MeV

Q: ARE LARGE n - systems unbound?

WHAT ABOUT NS?

→ BOUND BY GRAVITY!

→ DERIVE WEIZSÄCKER FORMULA w/ GRAVITY

(RECALL: IDENTICAL TO COULOMB
BUT: ATTRACTIVE

$$E_G = \frac{3\,GM^2}{5R} = \frac{3\,G m_n^2}{5 r_0} A^{5/3}$$

FOR CONST ρ

$$e = a_{vol} + a_{SF} A^{-1/3} + \frac{3}{5r_0}\left\{\frac{e^2}{4}(1-I)^2 - G m_n^2\right\} A$$

$$\left(a_{sym} + a_{ss} A^{-1/3}\right) I^2$$

AGAIN: large system, COULOMB REPULSI

SET $I = 1$

$$e \sim a_{vol} + a_{sym} - \frac{3Gm_n}{5r_0} A^{2/3}$$

for large A GRAV will eventually dominate

→ system will be bound

$$e < 0 \quad \text{for} \quad A > \left\{ \frac{5r_0}{3Gm_n^2}(a_{vol} + a_{sym}) \right\}^{3/2}$$

$$\simeq 0.79 \times 10^{56}$$

typical $A_{NS} \simeq 1.6 \times 10^{57}$

→ NS within a factor 20

Realistic MODEL

$$M_{min,NS} \sim 0.1\, M_\odot \sim 10^{56} \text{ neutrons}$$

But: 1) NS not pure n-matter:

β-decays to p-e pairs

2) central density $\rho_c \sim 4\rho_0$

→ μ (muons) and maybe free quarks at center?

3) treat in GR

Consequence: BINARY NS:

ONE ACCRETES, secondary eventually falls below lower limit

→ blow up!

→ Symmetry energy blows up star when grav. binding too low

$$e = \varepsilon_s(N+Z) + \varepsilon_A(N-Z)$$

Shell model

$\Leftrightarrow$ single particle levels

$\vec{j} = \vec{\ell} + \vec{s}$

$\rightarrow$ occupancy # $2j+1$

Substates $m_j = -j, \ldots j$

PARITY $\pi = (-1)^\ell$

Spectroscopic notation s, p, d, f, g...

for $\ell = 0, 1, 2, 3, 4 \ldots$

Spin-orbit term

$j = \ell + 1/2$ or $j = \ell - \frac{1}{2}$

ℓ integer $\rightarrow j$ ½-integer

here: very strong spin-orbit coupling

$\rightarrow$ significant change of single-particle states!

RESULT:

FORMATION OF E GAPS $\Rightarrow$ SHELLS.

$\rightarrow$ MAGIC #s: 2, 8, 20, 28, 50, 82, 128